CONFIDENCE INTERVAL CONFIDENCE PROBABILITY.

Bukhara Engineering-Technology Institute academic lyceum
Mathematics teacher of the "Exact Sciences" department
Ramazonov Jasurjon Rakhimovich
Bukhara Engineering-Technology Institute academic lyceum
Mathematics teacher of the "Exact Sciences" department
Yaxshiyev G'ayrat Erkinovich

Abstract: In order for us to derive the distribution law of some random variable X , we need to have a sufficiently large number of experiments and statistical material. In practice, it is often necessary to work with a limited amount of data.

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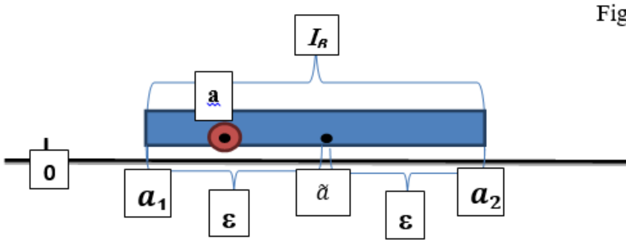
We considered the problem of estimating the unknown parameter a with 1 number. Such an estimate ("discrete ") is called a "point" estimate. In a number of problems, in addition to finding a suitable numerical value for the parameter α, it is also necessary to estimate the accuracy and reliability of α. What are the possible errors when replacing the parameter α $\tilde{a}$with a point estimate, and with what confidence can we say that these errors do not exceed certain limits. These types of issues are particularly important in small number of observations. $\tilde{a}$the point estimate is random, and substituting a $\tilde{a}$for would lead to serious errors. $\tilde{a}$in mathematical statistics, they use the concepts of confidence interval and confidence probability to express the reliability and accuracy of the estimate.

Let an unmixed estimate for the parameter a be obtained from the experiment . $\tilde{a}$We want to evaluate the possible error. Let's take a sufficiently large βprobability (For example: β= 0.9; 0.95 or 0.99). βLet the probability event occur in practice and εwe find such a number for which the following is valid: P(| $\tilde{a}$ – a | < ε) = βThen $\tilde{a}$the range of possible values of the error in converting a to ±εis; A large error in absolute value αoccurs with a small = 1 probability, and we write it in the following form:β

$P(\tilde{a} -$ ε < a < $\tilde{a}$ + ε) = β The value of the equality parameter a βprobably falls into the following interval: $I_\beta = (\tilde{a} -$ ε; $\tilde{a}$ + ε)

It is necessary to emphasize one thing. We looked at the probability of a random variable falling into a random non-random interval that occurs several times. Here the situation is different: the magnitude a is not random, I_β the interval is random. Its position on the abscissa axis is random, $\tilde{a}$determined by its center, and εthe length of the 2 interval is also random, since εthe magnitude is determined based on experiments. Therefore, in this case, βthe magnitude should not be considered as the probability that the point I_β "falls" into the interval, but rather I_β as the probability that the random interval covers the point a.

Figure 1



βA probability confidence $I_\beta$interval is called a confidence interval. $I_\beta$the limits of the interval : $a_1 = \tilde{a} -$ ε and $a_2 = \tilde{a}$ + ε called confidence limits.

We give another explanation to the concept of confidence interval. It (that is, the interval) can be considered as an interval of parameter values a that are consistent with the experimental data and do not negate them. In fact, αif we consider it as an event that happens with probability |$\tilde{a}$=1, the parameter a is β–a| the values that satisfy the >εcondition |$\tilde{a}$ − a | < ε can be considered compatible with them. $a_1$and $a_2$we move on to the problem of finding reliable limits. Let there be a nonmixed estimate for the parameter α . $\tilde{a}$If $\tilde{a}$the distribution law of magnitude were known, the problem of finding a confidence interval would be very simple. Now satisfying the following condition

P($|\tilde{a} - a| < \varepsilon$) = βit is enough to find the value. The problem is $\tilde{a}$that the distribution law of the estimate depends on the distribution law of the quantity x, and to get rid of its unknown difficulty, the following rough method can be used: εreplacing the unknown parameters in the expression for its point estimates. gives satisfactory results on

As an example, we consider the problem of confidence interval for mathematical expectation.

n random experiments were conducted on random variable *x* , *whose characteristics are mathematical expectation m and variance D - unknown.* The following estimates were obtained for these parameters.$\widetilde{D} = \frac{\sum_{i=1}^{n}(x_i - \bar{m})^2}{n-1}$ $\tilde{a} = \frac{\sum_{i=1}^{n} x_i}{n}$ It is necessary to construct a confidence interval corresponding to a reliable probability $I_\beta$for the mathematical expectation of the quantity βx . In solving this problem, $\widetilde{m}$the quantity is equal to the sum of n randomly distributed $x_i$random variables whose law is close to the law of normal distribution.

In practice, the distribution law of the sum can be considered as an approximate normal distribution even for relatively small additions (up to 10-20). $\bar{\text{L}}$et m be distributed according to the normal distribution law. The characteristics of this law are mathematical expectation and variance, respectively, m and $\frac{D}{n}$is equal to We assume that the quantity D is known to us and so $\varepsilon_\beta$we find the magnitude such that forP($|\widetilde{m} - \text{m}| < \varepsilon_\beta$) = β

appropriate $\Phi^*(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$formula, we express the probability on the left side of (1,3,5) by the normal function of the distribution:

$P(|\tilde{m} - m| < \varepsilon_\beta) = 2\,\Phi^*(\frac{\varepsilon_\beta}{\delta\tilde{m}}) - 1$here$\delta_{\tilde{m}} = \sqrt{\frac{D}{n}} - \tilde{m}$ the mean square deviation of the estimate $2\,\Phi^*\left(\frac{\varepsilon_\beta}{\delta\tilde{m}}\right) - 1 = \beta$of Eq $\varepsilon_\beta$find the value.

$\varepsilon_\beta = \delta\tilde{m}\ \arg\Phi^*(\frac{1+\beta}{2})$ Here, arg $\Phi^*(x)$ $\Phi^*$is the inverse function of the function (x), that is, the normal function of the distribution from this value of the argument is equal to x. $\delta_{\tilde{m}}$ The variance D is not clear to us, as its approximate value we $\tilde{D}$can use vaho and the following is appropriate.$\delta_{\tilde{m}} = \sqrt{\frac{\tilde{D}}{n}}$

Thus, the problem of constructing a confidence interval is approximately solved, and it is equal to:

$$I_\beta\ (\tilde{m} - \varepsilon_\beta\ ;\ m + \varepsilon_\beta)$$

here $\varepsilon_\beta$it is determined by the formula (1,3,7). When $\varepsilon_\beta$calculating $\Phi^*(x)$, it is convenient to build a table ((1,3,1) - see the table) βin which the values of the following quantity are entered.

$$t\beta = \arg\Phi^*(\frac{1+\beta}{2})$$

t gives the number of mean squared deviations for the normal distribution law, which should be placed to the right and left of the center βto be equal to the probability of falling into the resulting area . βThe confidence interval through the quantity t is expressed in the following formβ

$$I_\beta = (\tilde{m} - t\beta\ \delta_{\tilde{m}}\ ;\ \tilde{m} + t\beta\ \delta_{\tilde{m}})$$

table-1

β	tβ	β	tβ	β	tβ	β	tβ
0.80	1,282	0.86	1,475	0.91	1,694	0.97	2,169
0.81	1,310	0.97	1,613	0.92	1,750	0.99	2,325

0.82	1,340	0.88	1,554	0.93	1,810	0.99	2,576
0.83	1,371	0.89	1,597	0.94	1,880	0.9973	3,000
0.84	1,404	0.90	1,643	0.95	0.960	0.999	3,290
0.85	1,439			0.96	2,053		

The values of parameter m lying in this interval (1,3,2) are consistent with those given in the experiment given in the table. A confidence interval can also be constructed for a similar variance. Suppose that n random experiments are conducted on a random variable x with unknown parameters m and D, and a non-mixed estimate for the variance D is obtained.

$$\widetilde{D} = \frac{\sum_{i=1}^{n}(x_i-\widetilde{m})^2}{n-1}$$

$$\text{here}\widetilde{m} = \frac{\sum_{i=1}^{n} x_i}{n}$$

It is necessary to construct an approximate confidence interval for the variance. As can be seen from the formula (3.11), $\widetilde{D}$the magnitude $\frac{(x_i-\widetilde{m})^2}{n-1}$is equal to the sum of n random variables. This size is not arbitrary because each of them $\widetilde{m}$includes size and depends on others. As n increases, the distribution law of their sum approaches the normal distribution law. In practice, it can be considered normal when n = 20 30.÷
Let us assume that this is the case, and we will find the mathematical expectation and variance of the characteristics of this law. $\widetilde{D}$since the grade is not mixed, M[$\widetilde{D}$] = D
Let the variance be:

$$D[\widetilde{D}] = \frac{1}{n}M_4 - \frac{n-3}{n(n-1)}D^2$$

where $M_4$is the 4th central moment of magnitude x.

To use this expression, M_4it is necessary to put the (albeit approximate) values of D into it. D's $\widetilde{D}$estimate can be used instead. $M_4$4- the central moment can be replaced by its value, i.e. by the quantity in the following form.

$$M_4^* = \frac{\sum_{i=1}^{n}(x_i - \widetilde{m})^4}{n}$$

But such substitutions give not very high accuracy, because in a limited number of experiments, higher-order moments are determined with large errors. But in practice, the appearance of the distribution law of the quantity x is known in advance, and its parameters are unknown. Then M_4can be expressed by D. Let the quantity x be exponentially distributed. At that time, the 4th central moment is expressed by dispersion:

$$D[\widetilde{D}\ ] = \frac{2}{n-1}D^2$$

the unknown D in (1,3,14) with its estimate $\widetilde{D}$, we get:

$$\mathrm{D}[\widetilde{D}\ ] = \frac{2}{n-1}D^2$$

from this$\delta\widetilde{D} = \sqrt{\frac{2}{n-1}}\widetilde{D}$ M_4can also be expressed by D when the distribution of the quantity x is known but not normal. For example, for the flat density law, we have: Here (αβ) is the interval for which the law is given. From this$M_4 = 1{,}8\, D^2$

Based on the formula (3,12), we get the following:$\mathrm{D}[\widetilde{D}\ ] = \frac{0{,}8n+1{,}2}{n(n-1)}D^2$

In it we find approximately the following: $\delta\widetilde{D} = \sqrt{\frac{0{,}8n+1{,}2}{n(n-1)}}\widetilde{D}$It is recommended to use the formula (1,3,16) of the amount of X. $\delta\widetilde{D}$If an approximation for

$I_\beta = (\widetilde{D} - t_\beta\,\delta\widetilde{D}\ ;\ \widetilde{D} + t_\beta\,\delta\widetilde{D})$Here, $t_\beta$the magnitude is found from the given βprobability table

Summary:

Numerical characteristics of random variables, including mathematical expectation, variance and estimates for them, confidence interval, finding a confidence interval were studied.

REFERENCES USED

1. Е. С Вентцелъ. Теория вероятностей. Учеб для вузов – ъеизд. стер-м 1999

2.Бочаров П. П., Печинкин А. В. Теория вероятностей. Математическая статистика. - 2-е изд. - М.: ФИЗМАТЛИТ, 2005.

3. Ватутин В.А., Ивченко Г.И., Медведев Ю.И., Чистяков В.П. Теория вероятностей и математическая статистика в задачах М.: 2003.

4. Ивченко Г.И., Медведев Ю.И. Математическая статистика. М.: Высшая школа, 1984.