Preparation for solving trigonometric inequalities.

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To solve trigonometric inequalities, we need to master the main properties of basic trigonometric functions (sinx, cosx, tgx, ctgx) This includes their intervals and extremums, where the domain of definition and values, evenness, periodicity, zero-monotonicity intervals are stored. Here are the main properties for each of the basic trigonometric functions.

y = **sinx function.**

a) domain is all numbers in the set of real numbers

b) the set of values is E(sinx)=[-1;1] and is bounded.

c) The sine function is periodic with the smallest positive period 2p, for all x€R

$$\sin(x+2\pi)=\sin x$$

v)sine is an odd function sin(-x)=-sinx for all x €R

g) zeros sinx=0 x= pn

e) sinx>0 for all x€(2pn ; p+ 2pn)

j) sinx<0 for all x€(p+ 2pn; 2p+2pn) n€Z

k) growth interval x€[-p/2+2pn; When p/2+2pn] changes, the sine increases to -1.

m) decreasing interval`I x€[p/2+2pn; 3p/2+2pn] changes, the sine decreases from 1 to -1.

t) the maximum value of sine 1 reaches its maximum value at the point x=p/2+2pn.

l) the smallest value is sine -1. reaches its smallest value at the point x= $3\pi/2+2\pi n$.

y= cosx function

d) domain is all numbers in the set of real numbers
e) the set of values is E(cosx)=[-1;1] and is bounded.
f) cosine function periodic smallest positive period 2p, for all x€R
$\cos(x+2\pi)=\cos x$

v) cosine even function cos(-x)=cosx for all x €R

g) zeros cosx=0 x= p/2+pn

e) cosx>0 for all x€(-p/2+2pn; p/2+2pn)

j) cosx<0 for all x€(p/2+2pn; 3p/2+2pn) n€Z

k) growth interval x€[-p+2pn; 2pn] changes, cosine increases from -1 to 1.

m) decrease interval`I x€[2pn; p+2pn] changes, cosine decreases to -1.

t) the maximum value of cosine 1 reaches its maximum value at the point x=2pn.

l) the smallest value is sine -1. reaches its smallest value at the point x= p+2pn.

y=tgx is a function

a) detection area x= p/2+pn; Real numbers starting from the numbers in the form

defined in the collection.

b) the set of values is the set of all real numbers E(tgx)=R, so tgx

the function is unbounded

c) tg(-x)=-tgx for x in the field of definition of a corresponding odd function

g) the corresponding function is periodic with the smallest positive period tg(x+ p)=tgx for all x in the definition of p

f) zeros tgx=0 x= pn

g) tgx>0 all x€(pn; p/2+pn) tangents are positive in I and III quadrants
h) tgx<0 all x€(-p/2+pn; pn) tangents are negative in II and IV quadrants
i) As the growth interval I x€(-p/2+pn; p/2+pn) changes, the tangent increases at -∞ and +∞
j) Decreasing interval: no
k) Find the maximum and minimum values of the tangent function

Y=ctgx is a function

a) detection area x= pn; Real numbers starting from the numbers in the form

defined in the collection.

b) the set of values is the set of all real numbers E(ctgx)=R means ctgx

the function is unbounded

v) ctg(-x)=-ctgx for x in the field of definition of an odd function ctg

g) the cotangent function is periodic with the smallest positive period ctg(x+ p)=ctgx for all x in the definition of p

f) zeros ctgx=0 x= p/2+pn ;

l) ctgx>0 all x€(pn; p/2+pn) cotangis is positive in quadrants I and III
m) ctgx<0 all x€(-p/2+pn; pn) tangents are negative in quadrants II and IV
n) The growth interval is good
o) Decrease interval: ; (p n p+pn); decreases from -∞ to + ∞;

Since the cotangis function is not bounded, it has a maximum and a minimum value;

The intervals where the signs of the trigonometric functions are stored are respectively sinx>0, sinx<0, cosx>0, cosx<0, tgx>0 , tgx<0, ctgx>0, ctgz <0 are solutions of these inequalities.

Trigonometric related to the sine function

Solving inequalities

The simplest trigonometric inequalities related to the sine function

Sinx>a sinx≥a (1.3, 1)

Sinx<a and sinx≤a

It can be one of the views. (|a| ≤ 1)

Its properties are mainly used in solving trigonometric inequalities.

When solving inequalities in school textbooks, it is solved using its graphs and the unit circle. We will analyze the options in both ways.

I. sinx ≥a (1.3, 2)

If a>1 from (1.3, 2), the inequality has no solution.

If a=1, the roots of the equation sinx=1 are x= p/2+2pn; (1.3, 2) will be a solution of the inequality

If a ≤1, sinx ≥-1 is valid for all values of x.The following values of a need to be studied remain. 0<a≤ 1 or -1 <a≤0 sinx>a for the first case and sinx>-a for the second. We are dealing with solving sinx>0 in case I. Let's use the previous graphic methods. We draw a horizontal straight line y=a from the upper part of the OX axis. The x's that satisfy the values of the function lying above the graph y=a are the solutions of the inequality sinx>a

Since the period of the sinx function is 2p, it is enough to look at these inequalities at a distance of 2p and it is possible to find all the remaining solutions by adding 2pn to the obtained solution.

As can be seen from the graph, it is better to take from 0 to 2 p for such an interval. In this interval, the solution is simply written as follows. arcsinx1<x<arcsinx2 general solution of the given sinx>a inequality 2pn_arcsinx1<x<arcsinx2+2 pn This notation is a solution of the given inequality in some interval for each value of n€Z. For example: solve the inequality sinx>1/2. Draw the graphs of the functions y= sinx and y=1/2. (Chart 2) Our function y=sinx[0 p] intersects with the line y=1/2 in two places, or its sine value is equal to ½ in two places. plot is an illustration of the sinx>1/2 inequality we need values of x such that the value of the sine is above the graph of the function y=1/2. These are x1= arcsin ½= p/6 and x2 = p-arcsin1/2 = p- p/6=5 p/6

therefore, the solution of the inequality in the section [0;2 p] is written as follows.

p/6<x<5 p/6 the general solution is written as follows: 2 pn+ p/6<x<5 p/6+2 pn

This rule is kept even when the argument x is an expression.

For example: solve the inequality sin(2x+ p/4)>1/2. If we define z=2x+ p/4, then we will get to the inequality sinz>1/2 as before, and the solution solved for this variable will return to the previous variable.

The general solution of Sinz>1/2 is 2 pn+ p/6<z<5 p/6+2 pn, the solution of the inequality we need is written in the following form:2pn+ p/6<2x+ p /4<5 p/6+2 pn We solve this double inequality. Before all sides of this inequality we add p/4; 2pn+ p/6- p /4<2x<5 p/6- p /4+2 pn - p/12 +2 pn <2x<7 p/12+ 2pn. We find the general solution by dividing all terms by 2.p/24+ pn<x<7 p2/4+ pn

we will see how to find the solution for this case (sinx>a a>0) using the unit circle.

We mark the point with coordinate (0;a) from the ordinate axis. From this point, we draw a straight line parallel to the Ax axis. We denote the points of intersection with the unit circle by A and C, respectively. The ordinates of these points are the same and equal to y. We connect points A and C with the coordinate origin. It can be seen that the sinx>a inequality is satisfied at all points of the arc ABC, except for the beginning and end points.

So, this inequality is valid in the corners of the turn until our radius OA coincides with OC. According to the definition, sinx is the ordinate of point A to the unit circle. The radius OA1 to such a point x_1= arcsine When turning the corner, we get or arcsine to the starting point of the arc ABC. Point C also has such an ordinate, and considering the equality of arcs A1, A and CA $_{1,}$ point C of the arc corresponds only to x_2 = p-.All these considerations are based on the fact that the function y=sinx [o; It follows that p] has two solutions in the interval.

So, considering that one of the arc points of ABC satisfies our inequality, we write its solution in this interval as follows.

arcsina <x< p-arcsina

to find all the solutions of the inequality, we add 2 pn cycles to the ends of this interval.

2pn+ arcsine<x< p- arcsine+2 pn

We will show this in a concrete example.

Example : Solve the inequality sinx>1/2.

We draw a unit circle and take ½ section from the ordinate axis and draw a line parallel to the OX axis from this point. We will do the rest with easier considerations.

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