INVARIANCE PRINCIPLE FOR POSITIVELY ASSOCIATED RANDOM VARIABLES

Ziyadullayeva F,
National University of Uzbekistan, Tashkent, Uzbekistan;
ziadullaevaf7@gmail.com.

Definition. A finite collection of random variables$\{X_j,1\le j\le n\}$is said to be positively associated if for every every choice of component wise nondecreasing functions h and g from R^n to R,

$$Cov(h(X_1,...,X_n),g(X_1,...,X_n))\ge 0$$

when ever it exists; an infinite collection of random variables $\{X_n, n\ge 1\}$ is said to be positively associated if every finite sub-collection is associated.

Let the sequence $\{X_n,n\ge 1\}$ be a sequence of random variables with $EX_n=0$, $EX^2_n<\infty$, $n\ge 1$. Let $S_0=0,S_n=\sum_{k=2}^{n} Xk$, $\sigma^2_n=ES^2_n$, $n\ge 1$.

Assume that $\sigma^2_n>0$, $n\ge 1$. Let $\{k_n, n\ge 0\}$ be an increasing sequence of real numbers such that $0=k_0<k_1<k_2<...$ (1)

and $\lim_{n\to\infty}\max_{1\le i\le n}(k_i-k_{i-1})/k_n=0$. (2)

Define $m(t)=\max\{i: k_i \le t\}$, $t\ge 0$, and

$$W_n(t)=S_{mn}(t)/\sigma_n,\quad t\in [0,1],\ n\ge 1, \qquad (3)$$

where $m_n(t)=m(tk_n)$. Consider the process

$$W*_n(t)=S[nt]/\sigma_n,\quad ,t\in [0,1]. \qquad (4)$$

When $k_n=n$, $n\geq1$ the processes defined by (3) and (4) are equivalent.

Theorem 1 (Newman and Wright (1981)). Let the sequence $\{X_n, n\geq1\}$ be a strictly stationary sequence of positively associated random variables with $EX_1=0$ and $EX^2_1<\infty$. If $0<\sigma^2=\mathrm{Var}(X_1)+2\sum_{k=2}^{\infty}\mathrm{Cov}(X_1,X_n)<\infty$, then, $W*_n{}^L\to W$ as $n\to\infty$, where W is a standard Wiener process.

An invariance principle for a non-stationary associated process has been studied by Birkel (1988).

Theorem 2 (Birkel (1988)). Let the sequence $\{Xn, n\geq1\}$ be a sequence of associated random variables with $E(X_n)=0$, $E(X^2_n)<\infty$ for $n\geq1$. Assume that

(i) $\lim_{n\to\infty}\sigma^{-2}_n E(U_{nk}U_{nl})=\min(k,l)$ for $k,l\geq1$and $U_{m,n}=S_{m+n}-S_m$; and

(ii) $\sigma^{-2}_n(S_{n+m}-S_n)^2$, $m\geq0$, $n\geq1$ is uniformly integrable.

Then $W*_nL\to W$ as $n\to\infty$.

Birkel's result was generalized by Matula and Rychlik (1990). They observed

that if $W*_n{}^L\to W$ as $n\to\infty$, then $\sigma^2_n=nh(n)$, (5)

where $h: R+\to R+$ is slowly varying. They proved an invariance principle for sequences $\{X_n,n\geq1\}$ which do not satisfy the condition (5).

Theorem 3. Let $X_n,n\in N$,be centred,square-integrable and associated random variables.The following statements are equivalent:

(a) The random variables X_n, $n\in N$, satisfy a Central Limit Theorem,and

$$1/\sigma^2_n E(S_{nm}S_{nk})\longrightarrow\min(m,k),\ m,k\in N.$$

(b) The random variables X_n, $n \in N$, satisfy the invariance principle in D[0,1].

Corollary. Let X_n, $n \in N$, be centred, square-integrable and associated random variables.Assume that $u(n) \to 0$, $u(1) < \infty$,

$$\forall \varepsilon > 0, \quad 1/\sigma^2_n \sum_{j=1}^{n} \int_{|X_j| > \varepsilon s_n} X_j^2 dP \to 0 \quad \text{and} \quad 1/n\, \sigma^2_n \longrightarrow \sigma^2 \in (0,\infty)$$

are satisfied. Then the random variables X_n, $n \in N$, verify the invariance principle

in D[0,1].

Theorem 4. Let X_n,$n \in N$,be centred and associated random variables. Assume that

$$\inf_{n \in N} 1/n\, \sigma^2_n > 0, \qquad 1/\sigma^2_n\, \sigma^2_{nk} \longrightarrow k,\ k \in N$$

$$u(n) = O(n^{-\theta}) \text{ for some } \theta > 0, \qquad \sup_{n \in N} E|X_n|^{r+\eta} < \infty \text{ for some } r > 2,\ \eta > 0,$$

are satisfied. Then the random variables X_n, $n \in N$, verify the invariance principle

in D[0,1].

References

1. Birkel, T., 1988. The invariance principle for associated processes. Stochastic processes and Apple. 57-71.
2. Newman, C.M., Wright, A.L., 1981. An invariance principle for certain dependent sequences. Ann. Probab. V9. No 4, 671-675.