SPLAYN FUNKSIYALARINING TUGUN NUQLARDA UZLUKSIZLIGINI TEKSHIRISH

Qurbonov Jaloliddin Sapar o'g'li

O'zbekiston Milliy Universiteti

jaloliddinqurbonov2@gmail.com

Annotatsiya:*Agar har bir [xi, xi+1] qismiy kesmadagi splaynning noma'lum parametrlari boshqa qismiy kesmalardagi splaynlarning noma'lum parametrlari bilan birgalikda aniqlansa, bunday splaynlar global splaynlar deyiladi. Global splaynlarda qismiy kesmalardagi splaynlarning noma'lum parametrlari chiziqli algebraik tenglamalar sistemasini haydash usuli bilan yechish vositasida topiladi.*

Kalit so'zlar: *splayn funksiyalar, global splaynlar, lokal interpolyatsiyalash, lokal splaynlar, interpolyatsiyon ko'phadlar, klassik interpolyatsiyalash.*

Checking the continuity of spline functions at nodes

Abstract: If the unknown parameters of the spline in each [xi, xi+1] partial section are determined in conjunction with the unknown parameters of the splines in other partial sections, such splines are called global splines. In global splines, the unknown parameters of splines in partial sections are found by solving a system of linear algebraic equations by the driving method.

Keywords: spline functions, global splines, local interpolation, local splines, interpolation polynomials, classic interpolation.

Lokal interpolyatsion kubik splayn funksiyalarni umumiy nuqtalar asosida lokal interpolyatsion kubik splaynlarni qurish

Klassik interpolyatsion ko'phadlar [a, b] kesmada qurilsa, splayn funksiyalar esa [a, b] kesmani n ta bo'laklarga bo'linib, bitta bo'lakchada quriladi.Klassik interpolyatsion ko'p hadlar [a, b] kesmada bitta ko'p had qurilib yaqinlashadi. Splayn funksiyalar esa [a, b] kesma n ta bo'lakga bo'lganda har bir bo‘lakda yaqinlashadi va butun [a, b] oraliqda yaqinlashishini ta’minlaydi. Splayn funksiyalar klassik interpolyatsion ko‘p hadlarga nisbatan qurilishi, tadbiq etilishi sodda va tiklanayotgan obektga tez yaqinlashishini ta’minlaydi.

Lokal interpolyatsion kubik splayn funksiya fan va texnikaning rivojlanishida ayniqsa amaliy masalalarga qo‘llash borosida dolzarb masalalardan hisoblanadi.

Xususan: xar xil turdagi geofizik, biomeditsina, ekologik jarayonlarda va boshqa sohalarda signallarni tiklash, qayta ishlash va splayn modellar asosida bashorat qilish masalalarida ko‘plab natijalar olinmoqda.

Undan tashqari lokal interpolyatsion kubik splayn funksiya asosida qurilgan kvadratur, va kubatur formulalar yordamida regulyar va singulyar integrallarni taqribiy xisoblashlarda hamda regulyar va singulyar integral tenglamalarni taqribiy echishda yaxshi natijalarga erishilmoqda.

*Lokal interpolyatsion kubik splayn funksiya xaqida qisqacha ma’lumot***.**

Ushbu $y_i(x)$ va $y_{i+1}(x)$ parabolalarning qurilishini qaraymiz:

$$(x_{i-1}, y_{i-1}),\ (x_i, y_i),\ (x_{i+1}, y_{i+1}) \qquad (1)$$

nuqtalardan o‘tuvchi

$$y_i(x) = a_i x^2 + b_i x + c_i \qquad (2)$$

Parabolani qurish uchun quyidagi tenglamalar sistemasini hosil qilamiz

$$\begin{aligned} a_i x^2_{i-1} + b_i x_{i-1} + c_i &= y_{i-1} \\ a_i x^2_i + b_i x_i + c_i &= y_i \\ a_i x^2_{i+1} + b_i x_{i+1} + c_i &= y_{i+1} \end{aligned} \qquad (3)$$

tenglamalar sistemasini echib, a_i, b_i, c_i larning qiymatlari topiladi va $y_i(x)-$ paraboladagi a_i, b_i, c_i lar o‘rniga qo‘yilib, $y_i(x)-$ parabolaning umumiy ko‘rinishi hosil qilinadi.

$$y_i(t) = -0.5t(1-t)y_{i-1} + (1-t)^2 y_i + 0.5t(1+t)y_{i+1}; \qquad (4)$$

bu erda $t = (x - x_i)/h;$ $x \in [x_i, x_{i+1}]$.

Endi quyidagi nuqtalardan o‘tuvchi

(x_i, y_i); (x_{i+1}, y_{i+1}); (x_{i+2}, y_{i+2}) (5)

nuqtalardan o‘tuvchi

$$y_{i+1}(x) = a_{i+1}x^2 + b_{i+1}x + c_{i+1} \quad (6)$$

Parabolani umumiy ko‘rinishini hosil qilish uchun quyidagi tenglamalar sistemasi echiladi

$$\begin{aligned} &a_i x^2{}_{i-1} + b_i x_{i-1} + c_i = y_{i-1} \\ &a_i x^2{}_i + b_i x_i + c_i = y_i \\ &a_i x^2{}_{i+1} + b_i x_{i+1} + c_i = y_{i+1} \end{aligned} \quad (7)$$

tenglamalar sistemasi echilib, a_{i+1}, b_{i+1}, c_{i+1} larning qiymatlari topilib, $y_{i+1}(x)-$ paraboladagi a_{i+1}, b_{i+1}, c_{i+1} koeffesienti o‘rniga qo‘yilib, $y_{i+1}(x)-$ parabolaning umumiy ko‘rinishi hosil qilinadi.

$$y_{i+1}(t) = 0.5(t^2 - 3t + 2)y_i - t(t-2)y_{i+1} - 0.5t(1-t)y_{i+2} \quad (8)$$

bu erda $t = (x - x_i)/h;$ $x \in [x_i, x_{i+1}]$.

Yuqoridagi qurilgan $y_{i+1}(x)$ va $y_i(x)$ parabolalarning quyidagi chiziqli

$$S_i(x) = S_i(t) = (\alpha_1 + \alpha_2 t)y_i(x) + (\alpha_3 + \alpha_4 t)y_{i+1}(x). \quad (9)$$

kombinatsiyasi asosida lokal interpolyatsion kubik splayn funksiya hosil qilinadi.

Ushbu splayn funksiyani qurilishida dastlab $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ parametrlarini topish kerak bo‘ladi.Buning uchun 4- ta nomalumli 4- ta tenglamalar sistemasi tuzilib, sistema echilib $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ lar topiladi va lokal interpolyatsion splayn funksiya quriladi. Tenglamalarning birinchi ikkitasi quyidagi

$$S_i(x_i) = f_i, \qquad S_i(x_{i+1}) = f_{i+1}$$

Intepolyatsiya shartlaridan foydalanib hosil qilinadi, ya’ni

$$\alpha_1 + \alpha_3 = 1, \qquad \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1,$$

bu tenglamalardan $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ noma’lumlarni topishda foydalanadigan quyidagi ikkita tenglamalarni

$$\alpha_3 = 1 - \alpha_1, \quad \alpha_4 = -\alpha_2$$

hosil qilamiz. $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ noma'lumlarni topishda foydalanadigan yana ikkita tenglamalarni topish uchun ikkita yonma yon oraliqlarda qo'rilgan $S_i(x), x \in [x_i; x_{i+1}]$ va $S_{i+1}(x), x \in [x_{i+1}; x_{i+2}]$ lokal interpolyatsion splayn funksiyalarning birinchi va ikkinchi tartibli hosilalarini olib $x = x_{i+1}$ tugun nuqtada uzluksizlik shartlarini bajarilishidan hosil qilinad.

Ya'ni

$$S'_i(x_{i+1}) = S'_{i+1}(x_{i+1})$$
$$S''_i(x_{i+1}) = S''_{i+1}(x_{i+1})$$

Ushbu $S_i(x)$ lokal interpolyatsion splayn funksiyani birinchi va ikkinchi tartibli hosilalarini olib $x = x_{i+1}$ tugun nuqtadagi qiymatlarini qo'yib, ma'lum bir ixchamlashlardan keyin quyidagi ko'rinishga keladi.

$$\alpha_1(\Delta^2 f_{i+1} - \Delta^2 f_{i-1}) + \alpha_2 \Delta^3 f_{i-1} = \Delta^3 f_i, \quad (10)$$

$$\alpha_1(\Delta^4 f_{i-1}) - \alpha_2(\Delta^2 f_{i-1} + \Delta f_i - \Delta f_{i+2}) = \Delta^3 f_i. \quad (11)$$

Bu erda Δ -chekli ayirmali operator.

Natijada $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ nomalumlarni topish uchun 4- ta noma'lumli 4- ta tenglamalar sistemasini hosil qildik, ya'ni

$$\begin{cases} \alpha_3 = 1 - \alpha_1 \\ \alpha_4 = -\alpha_2 \\ \alpha_1(\Delta^2 f_{i+1} - \Delta^2 f_{i-1}) + \alpha_2 \Delta^3 f_{i-1} = \Delta^3 f_i \\ \alpha_1(\Delta^4 f_{i-1}) - \alpha_2(\Delta^2 f_{i-1} + \Delta f_i - \Delta f_{i+2}) = \Delta^3 f_i. \end{cases} \quad (12)$$

Ushbu tenglamalar sistemasini echimlari mos ravishda $\alpha_1^*, \alpha_2^*, \alpha_3^*, \alpha_4^*$ bo'lsin desak ya'ni $\alpha_1 = \alpha_1^*;\ \alpha_2 = \alpha_2^*\ \ \alpha_3 = \alpha_3^*;\ \alpha_4 = \alpha_4^*$ topiladi. Natijada quyidagi

$$S_3(x) = S_3(t) = (\alpha_1^* + \alpha_2^* t) y_i(t) + (\alpha_3^* + \alpha_4^* t) y_{i+1}(t) \quad (13)$$

defekti 1 ga teng bo'lgan lokal interpolyatsion splayn funksiyani umumiy ko'rinishini topamiz.

Ammo $f_{i-1}, f_i, f_{i+1}, f_{i+2}, f_{i+3}$ koeffitsientlar berilgan murakkab ratsional funksiyalardir. Shu sababli, bunday splaynlar signallarga raqamli ishlov berish uchun noqulaydir.

Shu bois $\alpha_1 = 1,\ \alpha_2 = -1$ desak, u holda $\alpha_3 = 0,\ \alpha_4 = 1$ ga teng bo'ladi.

$$S_3(f;x) = S_3(t) = (1-t)y_i(t) + ty_{i+1}(t). \qquad (14)$$

Belgilashlar kiritib olamiz

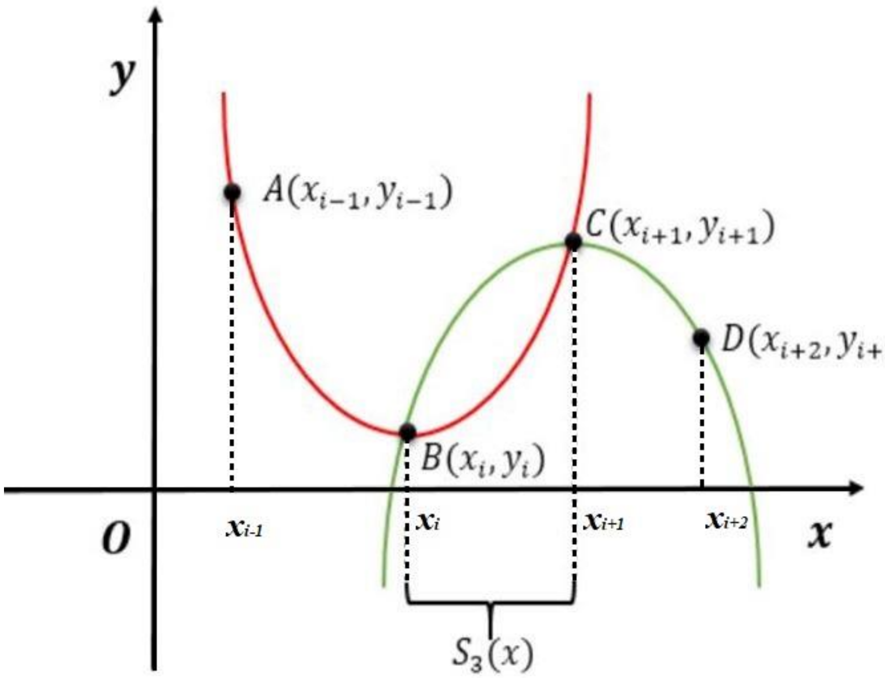
$$\varphi_1(t) = -0,5t(1-t)^2, \qquad \varphi_2(t) = 0,5(1-t)(2+2t-3t^2),$$

$$\varphi_3(t) = 0,5t(1+4t-3t^2), \qquad \varphi_4(t) = -0,5(1-t)t^2$$

(3.3.14) da berilgan $y_i(t)$ va $y_{i+1}(t)$ larning o'rniga (3.3.4), (3.3.8) ifodalarni qo'ysak, bizda $[x_i, x_{i+1}]$ oraliq uchun (3.3.15) ko'rinish hosil bo'ladi.

$$S_3(f;x) = \sum_{j=0}^{3} \varphi_{j+1}(t) f(x_{i+j-1}). \qquad (15)$$

Splaynlar nazaryatsiga asoslanib, ushbu $[x_i, x_{i+1}]$ oraliqdagi (3.3.15) funksiyani lokal interpolyatsion kubik splayn funksiya deb atash mumkin



1-rasm

Aniq berilgan ma'lumotlar asosida 2-ta umumiy nuqtalarga ega bo'lgan parabolik funksiyalarni qurish.

Soddalik uchun parabolalarnig qurilishida tugun nuqtalar $h=1$ qadam bilan olindi.

$Y_1(x)$, $Y_2(x)$, $Y_3(x)$, $Y_4(x)$, $Y_5(x)$, $Y_6(x)$, $Y_7(x)$, $Y_8(x)$ – parabolalarning grafiklarini qurishda quyidagi

$A(-1,2);\ B(0,1);\ C(1,2);\ D(2,1);\ E(3,1.5);\ F(4,0.5);$
$G(5,2.5);\ H(6,1.5);\ I(7,\ 2);\ K(8,\ 0.5)$

berilgan nuqtalardan foydalanilgan holda chizilgan.

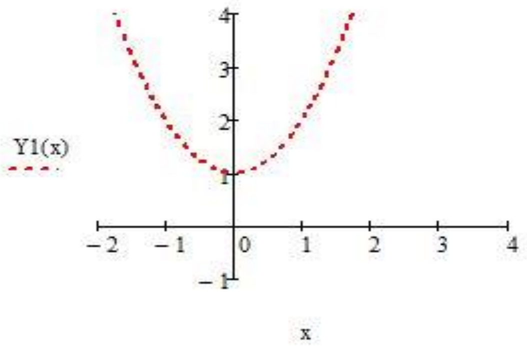
Qaralayotgan $Y_1(x)$-parabola $A(-1,2);\ B(0,1);\ C(1,2)$- nuqtalardan o'tuvchi paraboladir va bu $Y_1(x)$ - parabola quyidagi ko'rinishga ega bo'ladi.

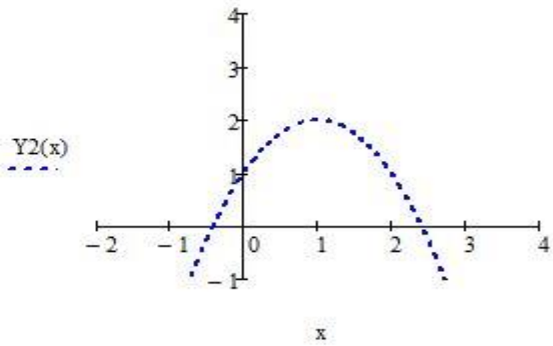
$Y_1(x)=x^2+1$

$Y_2(x)$-parabola $B(0,1);\ C(1,2);\ D(2,1)$ - nuqtalardan o'tuvchi paraboladir. $Y_3(x)$-parabola $C(1,2);\ D(2,1);\ E(3,1.5)$ - nuqtalardan o'tuvchi paraboladir. $Y_4(x)$-parabola $D(2,1);\ E(3,1.5);\ F(4,0.5)$ - nuqtalardan o'tuvchi paraboladir. $Y_5(x)$ - parabola $E(3,1.5);\ F(4,0.5);\ G(5,2.5)$ - nuqtalardan o'tuvchi paraboladir. $Y_6(x)$ - parabola $F(4,0.5);\ G(5,2.5);\ H(6,1.5)$ - nuqtalardan o'tuvchi paraboladir. $Y_7(x)$-parabola $G(5,2.5);\ H(6,1.5);\ I(7,\ 2)$ -nuqtalardan o'tuvchi paraboladir. $Y_8(x)$-parabola $H(6,1.5);\ I(7,\ 2);\ K(8,\ 0.5)$ - nuqtalardan o'tuvchi paraboladir. $Y_1(x)$, $Y_2(x)$, $Y_3(x)$, $Y_4(x)$, $Y_5(x)$, $Y_6(x)$, $Y_7(x)$, $Y_8(x)$ - parabolalarning grafiklarini chizish uchun Mathcad paketidan foydalanildi.

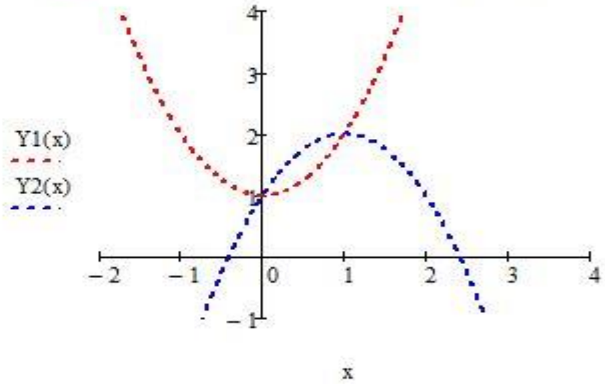
2-rasm

$Y1(x) := x^2 + 1 \quad x \in [0,1] \quad h := 1$ $\qquad$ $Y2(x) := -x^2 + 2x + 1 \quad x \in [0,1] \quad h := 1$

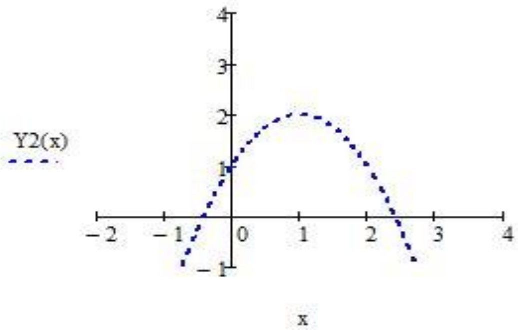


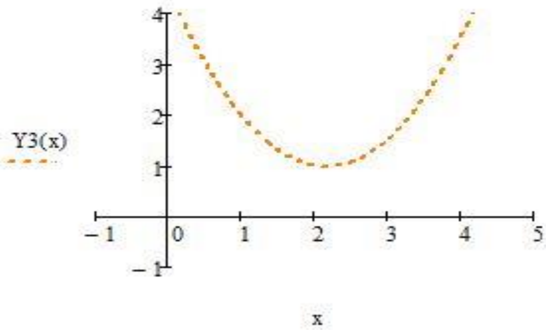


$Y1(x) := x^2 + 1 \quad Y2(x) := -x^2 + 2x + 1 \quad x \in [0,1] \quad h := 1$

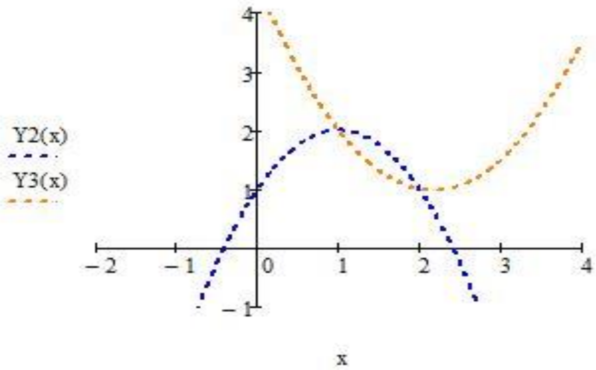


$Y2(x) := -x^2 + 2x + 1 \quad x \in [1,2] \quad h := 1$ $\qquad$ $Y3(x) := \frac{3}{4}x^2 - \frac{13}{4}x + 4.5 \quad x \in [1,2] \quad h := 1$



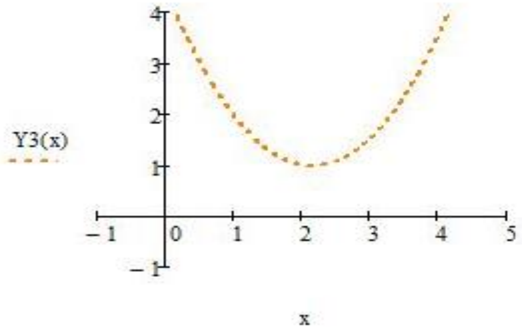


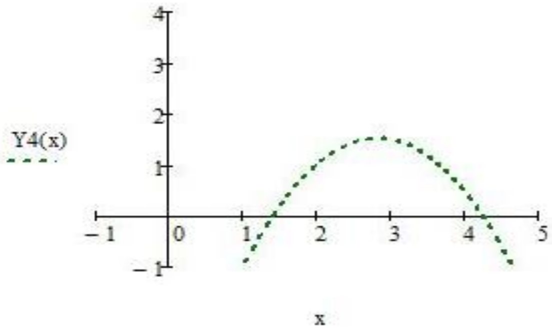
$Y2(x) := -x^2 + 2x + 1 \quad Y3(x) := \frac{3}{4}x^2 - \frac{13}{4}x + 4.5 \quad x \in [1,2] \quad h := 1$



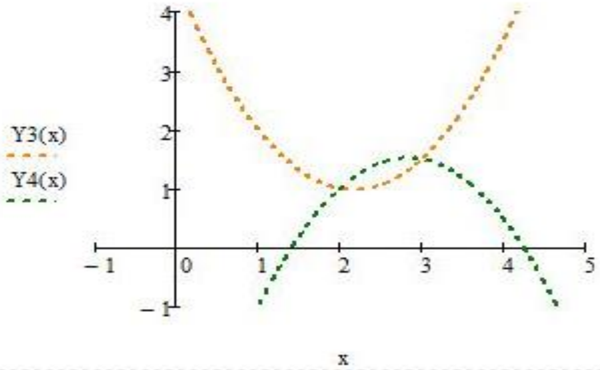
ISSN 3060-4567

$Y3(x) := \frac{3}{4}x^2 - \frac{13}{4}x + 4.5 \quad x \in [2,3] \quad h := 1$ $\qquad Y4(x) := \frac{-3}{4}x^2 + \frac{17}{4}x - 4.5 \quad x \in [2,3] \quad h := 1$

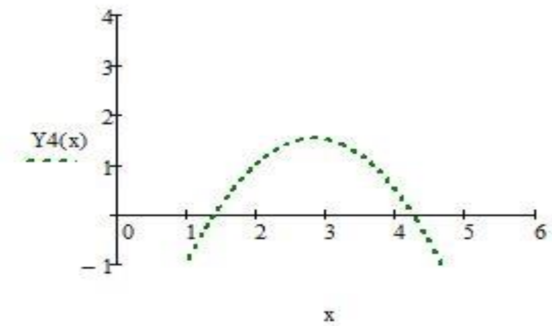


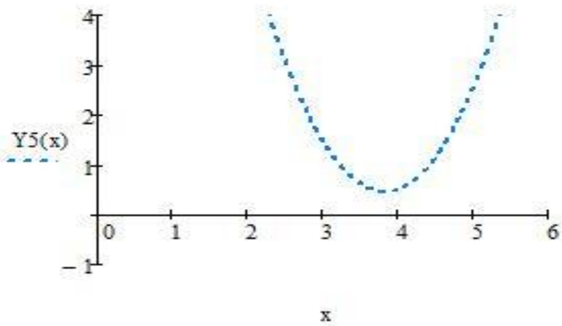


$Y3(x) := \frac{3}{4}x^2 - \frac{13}{4}x + 4.5 \qquad Y4(x) := \frac{-3}{4}x^2 + \frac{17}{4}x - 4.5 \qquad x \in [2,3] \qquad h := 1$

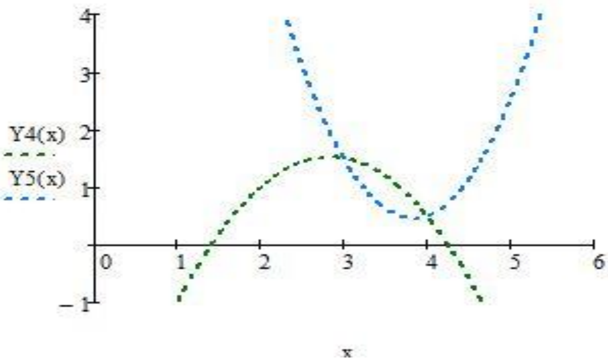


$Y4(x) := \frac{-3}{4}x^2 + \frac{17}{4}x - 4.5 \quad x \in [3,4] \quad h := 1$ $\qquad Y5(x) := \frac{3}{2}x^2 - \frac{23}{2}x + 22.5 \quad x \in [3,4] \quad h := 1$



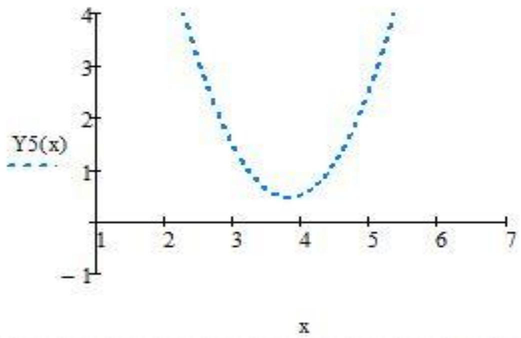


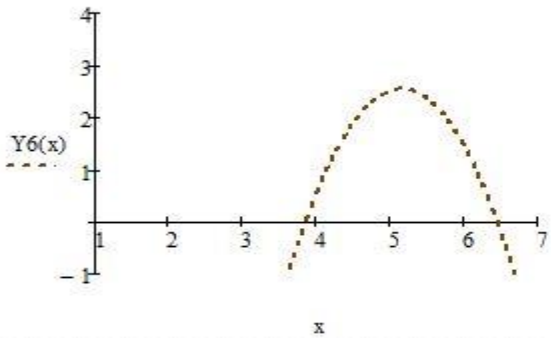
$Y4(x) := \frac{-3}{4}x^2 + \frac{17}{4}x - 4.5 \qquad Y5(x) := \frac{3}{2}x^2 - \frac{23}{2}x + 22.5 \qquad x \in [3,4] \qquad h := 1$



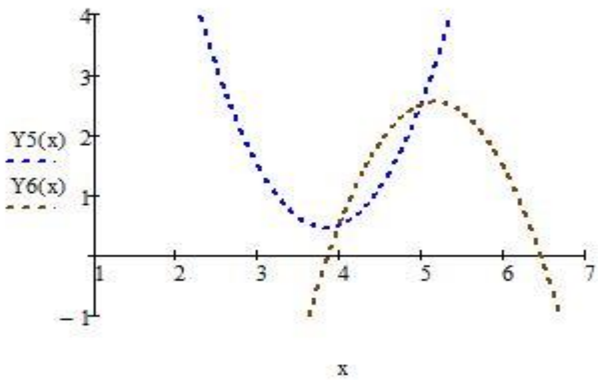
ISSN 3060-4567

$Y5(x) := \frac{3}{2}x^2 - \frac{23}{2}x + 22.5 \quad x \in [4,5] \quad h := 1$ $\qquad Y6(x) := \frac{-3}{2}x^2 + 15.5x - \frac{75}{2} \quad x \in [4,5] \quad h := 1$

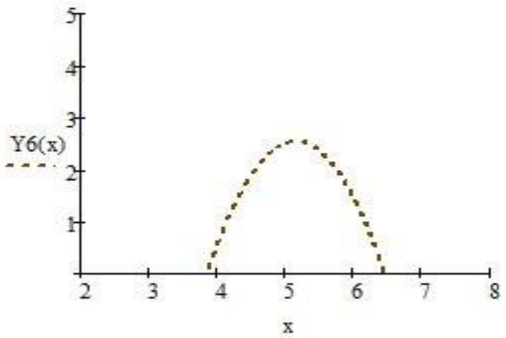


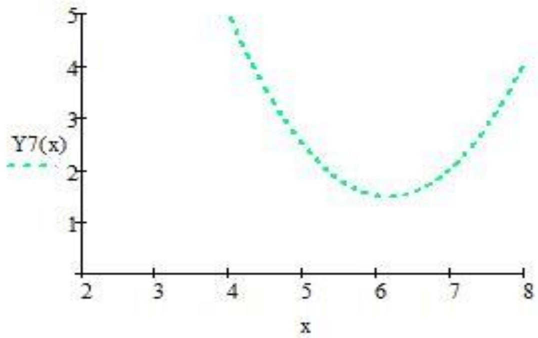


$Y5(x) := \frac{3}{2}x^2 - \frac{23}{2}x + 22.5 \qquad Y6(x) := \frac{-3}{2}x^2 + 15.5x - \frac{75}{2} \qquad x \in [4,5] \qquad h := 1$

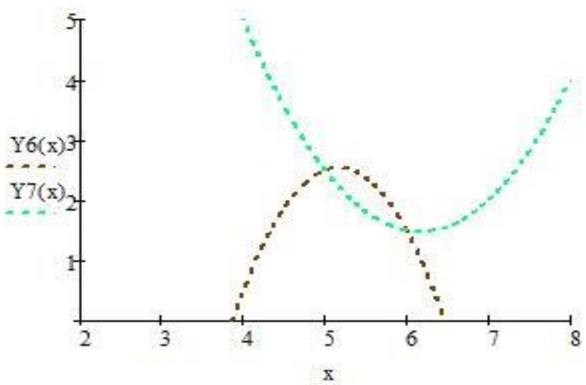


$Y6(x) := \frac{-3}{2}x^2 + 15.5x - \frac{75}{2} \quad x \in [5,6] \quad h := 1$ $\qquad Y7(x) := \frac{3}{4}x^2 - \frac{37}{4}x + 30 \quad x \in [5,6] \quad h := 1$

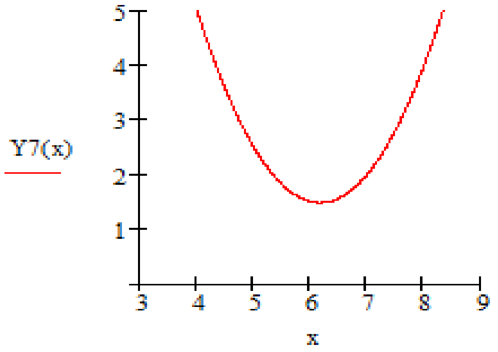


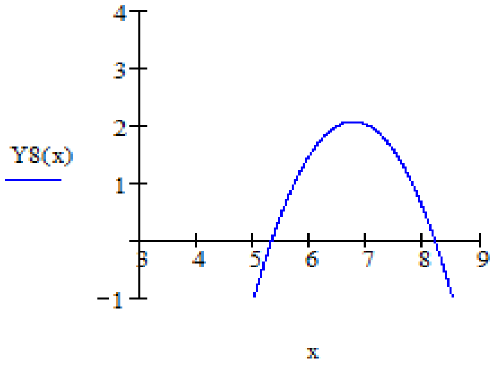


$Y6(x) := \frac{-3}{2}x^2 + 15.5x - \frac{75}{2} \qquad Y7(x) := \frac{3}{4}x^2 - \frac{37}{4}x + 30 \qquad x \in [5,6] \qquad h := 1$

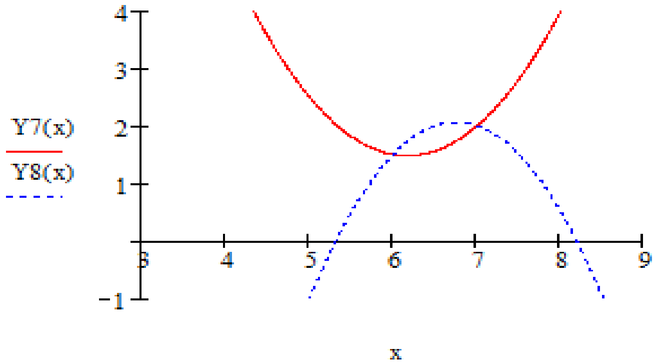


$Y7(x) := \frac{3}{4}x^2 - \frac{37}{4}x + 30$ $\qquad$ $Y8(x) := -x^2 + \frac{27}{2}x - 43.5$



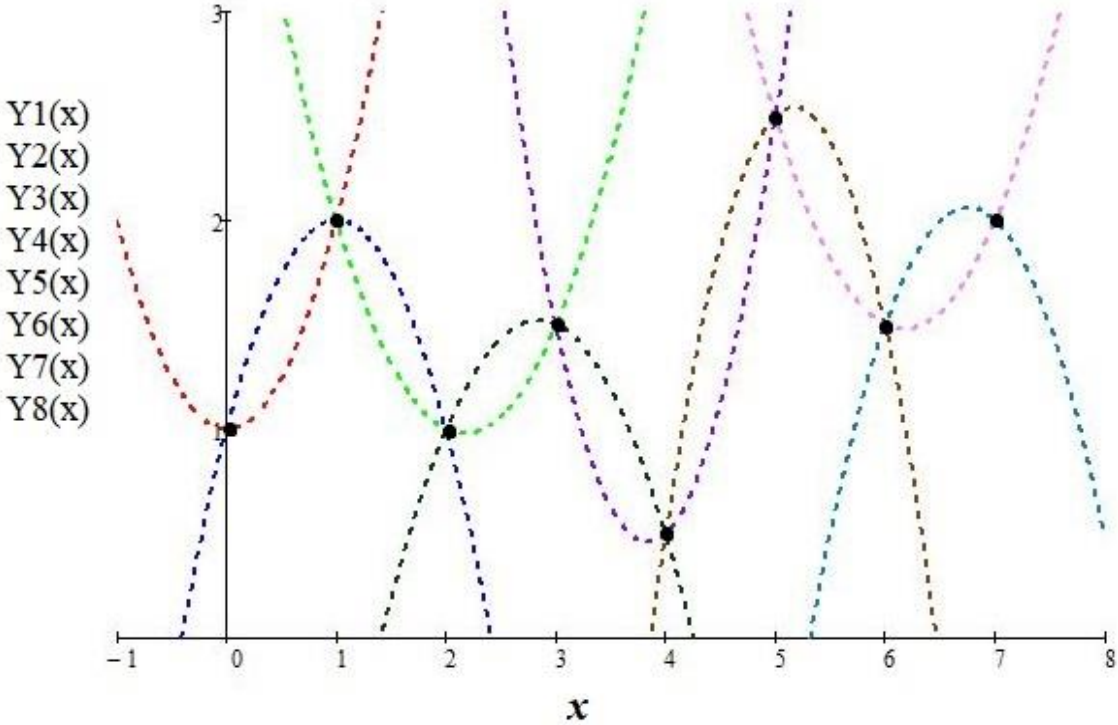


$Y7(x) := \frac{3}{4}x^2 - \frac{37}{4}x + 30$ $\qquad$ $Y8(x) := -x^2 + \frac{27}{2}x - 43.5$



Yuqorida qaralgan parabolik funksiyalarni chiziqli kombinatsiyalari asosida lokal interpolyatsion kubik splayn funksiyalarni quramiz.

Yuqoridagi $Y_1(x)$, $Y_2(x)$, $Y_3(x)$, $Y_4(x)$, $Y_5(x)$, $Y_6(x)$, $Y_7(x)$, $Y_8(x)$ – parabolalarning chiziqli kombinatsiyasi (9) asosida 3-darajali $S_3(x)$ lokal interpolyatsion kubik-splayn funksiyalar hosil bo‘ladi. Endi ushbu parabolalarni grafiklarini bitta Dekart koordinatalar sistemasida chizamiz. Bu erda har bir yonma-yon turgan parabolalar ikkitadan umumiy nuqtalarga ega bo‘ladi. Bu holatni parabolalarning grafiklaridan ham ko‘rish mumkin.



3-rasm

Lokal interpolyatsion kubik splayn funksiyalarni ulanish tugun nuqtalarida aniq berilgan ma'lumotlar asosida uzluksizligini tekshirish

Ushbu bo'limda biz bundan oldingi bo'limda qurilgan parabolik funksiyalarni chiziqli kombinatsiyalari asosida lokal interpolyatsion kubik splayn funksiyalarni qurilish jarayonini tahlil qilamiz. Buning uchun biz qurgan $Y_i(x)$ larni quyidagi oraliqlarda chiziqli kombinatsiyasi asosida S(x) larni quramiz va ulanish tugun nuqtalaridagi uzluksizligini tekshiramiz.

Dastlab yuqoridagi qurilgan $Y_1(x)$ va $Y_2(x)$ parabolalarning chiziqli kombinatsiyasi asosida $S_{3(1)}(x)$, $x \in [0,\ 1]$ lokal interpolyatsion kubik splayn funksiya qurildi.

Bu erda: $Y_1(x) = x^2 + 1$ va $Y_2(x) = -x^2 + 2x + 1$;

$S_{3(i)}(t) = (1-t)Y_i(t) + tY_{i+1}(t)$, $t = (x - x_i)/h$; $x \in [x_i, x_{i+1}]$.

Bizga ma'lumki $x_i = 0$, $h = 1$, $x \in [0,1]$. ekanligidan foydalanib

$t = \dfrac{x - x_i}{h} = \dfrac{x - 0}{1} = x$; yani: $t = x$ keltirib chiqaramiz va (9) ga oborib qo'yamiz. Natijada ushbu ifoda xosil bo'ladi:

$$S_{3(1)}(x)=(1-x)\cdot Y_1(x)+x\cdot Y_2(x)=(1-x)(x^2+1)+x(-x^2+2x+1)=$$
$$x^2+1-x^3-x-x^3+2x^2+x=-2x^3+3x^2+1.$$
$$S_{3(1)}(x)=-2x^3+3x^2+1.$$

2. Yuqoridagi kabi qurilgan $Y_2(x)$ va $Y_3(x)$ parabolalarning chiziqli kombinatsiyasi asosida $S_{3(2)}(x)$, $x\in[1,\ 2]$ lokal interpolyatsion kubik splayn funksiya qurildi.

Bu erda: $Y_2(x)=-x^2+2x+1$ va

$$Y_3(x)=\frac{3}{4}x^2-\frac{13}{4}x+4.5;$$

$$S_{3(i)}(t)=(1-t)Y_i(t)+tY_{i+1}(t),\quad t=(x-x_i)/h;\quad x\in[x_i,x_{i+1}].$$

Bizga ma'lumki $x_i=0,\quad h=1,\quad x\in[1,2]$. ekanligidan foydalanib $t=\frac{x-x_i}{h}=\frac{x-0}{1}=x;$ yani: $t=x$ keltirib chiqaramiz va (9) ga oborib qo'yamiz. Natijada ushbu ifoda xosil bo'ladi:

$$S_{3(2)}(x)=(1-x)\cdot Y_2(x)+x\cdot Y_3(x)=(1-x)(-x^2+2x+1)+x(\frac{3}{4}x^2-\frac{13}{4}x+4.5)=$$
$$-x^2+2x+1+x^3-2x^2-x+\frac{3}{4}x^3-\frac{13}{4}x^2+4.5x=\frac{7}{4}x^3-\frac{25}{4}x^2+\frac{11}{2}x+1.$$
$$S_{3(2)}(x)=\frac{7}{4}x^3-\frac{25}{4}x^2+\frac{11}{2}x+1$$

3. Yuqoridagi kabi qurilgan $Y_3(x)$ va $Y_4(x)$ parabolalarning chiziqli kombinatsiyasi asosida $S_{3(3)}(x)$, $x\in[2,\ 3]$ lokal interpolyatsion kubik splayn funksiya qurildi.

Bu erda: $Y_3(x)=\frac{3}{4}x^2-\frac{13}{4}x+4.5$ va

$$Y_4(x)=-\frac{3}{4}x^2+\frac{17}{4}x-4.5;$$

$$S_{3(i)}(t)=(1-t)Y_i(t)+tY_{i+1}(t), \quad t=(x-x_i)/h; \qquad x\in[x_i,x_{i+1}].$$

Bizga ma'lumki $x_i=0, \quad h=1, \quad x\in[2,3]$. ekanligidan foydalanib

$t=\frac{x-x_i}{h}=\frac{x-0}{1}=x;$ yani: $t=x$ keltirib chiqaramiz va (9) ga oborib qo'yamiz. Natijada ushbu ifoda xosil bo'ladi:

$$S_{3(3)}(x)=(1-x)\cdot Y_3(x)+x\cdot Y_4(x)=(1-x)(\frac{3}{4}x^2-\frac{13}{4}x+4.5)+x(-\frac{3}{4}x^2+\frac{17}{4}x-4.5)=$$

$$\frac{3}{4}x^2-\frac{13}{4}x+4.5-\frac{3}{4}x^3+\frac{13}{4}x^2-4.5x-\frac{3}{4}x^3+\frac{17}{4}x^2-4.5x=-\frac{3}{2}x^3+\frac{33}{4}x^2-\frac{49}{4}x+\frac{9}{2}.$$

$$S_{3(3)}(x)=-\frac{3}{2}x^3+\frac{33}{4}x^2-\frac{49}{4}x+\frac{9}{2}.$$

4. Yuqoridagi kabi qurilgan $Y_4(x)$ va $Y_5(x)$ parabolalarning chiziqli kombinatsiyasi asosida $S_{3(4)}(x)$, $x\in[3,\ 4]$ lokal interpolyatsion kubik splayn funksiya qurildi.

Bu erda: $Y_4(x)=-\frac{3}{4}x^2+\frac{17}{4}x-4.5$ va

$Y_5(x)=\frac{3}{2}x^2-\frac{23}{2}x+22.5;$

$$S_{3(i)}(t)=(1-t)Y_i(t)+tY_{i+1}(t), \quad t=(x-x_i)/h; \qquad x\in[x_i,x_{i+1}].$$

Bizga ma'lumki $x_i=0, \quad h=1, \quad x\in[3,4]$. ekanligidan foydalanib

$t=\frac{x-x_i}{h}=\frac{x-0}{1}=x;$ yani: $t=x$ keltirib chiqaramiz va (3.2.9) ga oborib qo'yamiz. Natijada ushbu ifoda xosil bo'ladi:

$$S_{3(4)}(x)=(1-x)\cdot Y_4(x)+x\cdot Y_5(x)=(1-x)(-\frac{3}{4}x^2+\frac{17}{4}x-4.5)+x(\frac{3}{2}x^2-\frac{23}{2}x+22.5)=$$

$$-\frac{3}{4}x^2+\frac{17}{4}x-4.5+\frac{3}{4}x^3-\frac{17}{4}x^2+4.5x+\frac{3}{2}x^3-\frac{23}{2}x^2+22.5x=\frac{9}{4}x^3-\frac{33}{2}x^2+\frac{125}{4}x-\frac{9}{2}.$$

$$S_{3(4)}(x)=\frac{9}{4}x^3-\frac{33}{2}x^2+\frac{125}{4}x-\frac{9}{2}.$$

ISSN 3060-4567

5. Yuqoridagi kabi qurilgan $Y_5(x)$ va $Y_6(x)$ parabolalarning chiziqli kombinatsiyasi asosida $S_{3(5)}(x)$, $x\in[4,\ 5]$ lokal interpolyatsion kubik splayn funksiya qurildi.

Bu erda: $Y_5(x)=\frac{3}{2}x^2-\frac{23}{2}x+22.5$ va

$Y_6(x)=-\frac{3}{2}x^2+\frac{31}{2}x-\frac{75}{2}$;

$$S_{3(i)}(t)=(1-t)Y_i(t)+tY_{i+1}(t),\quad t=(x-x_i)/h;\qquad x\in[x_i,x_{i+1}].$$

Bizga ma'lumki $x_i=0,\quad h=1,\quad x\in[4,5]$. ekanligidan foydalanib

$t=\frac{x-x_i}{h}=\frac{x-0}{1}=x$; yani: $t=x$ keltirib chiqaramiz va (9) ga oborib qo'yamiz. Natijada ushbu ifoda xosil bo'ladi:

$$S_{3(5)}(x)=(1-x)\cdot Y_5(x)+x\cdot Y_6(x)=(1-x)(\frac{3}{2}x^2-\frac{23}{2}x+22.5)+x(-\frac{3}{2}x^2+\frac{31}{2}x-\frac{75}{2})=$$

$$\frac{3}{2}x^2-\frac{23}{2}x+22.5-\frac{3}{2}x^3+\frac{23}{2}x^2-22.5x-\frac{3}{2}x^3+\frac{31}{2}x^2-\frac{75}{2}x=-3x^3-\frac{57}{2}x^2-\frac{143}{2}x+\frac{45}{2}.$$

$$S_{3(5)}(x)=-3x^3-\frac{57}{2}x^2-\frac{143}{2}x+\frac{45}{2}.$$

6. Yuqoridagi kabi qurilgan $Y_6(x)$ va $Y_7(x)$ parabolalarning chiziqli kombinatsiyasi asosida $S_{3(6)}(x)$, $x\in[5,\ 6]$ lokal interpolyatsion kubik splayn funksiya qurildi.

Bu erda: $Y_6(x)=-\frac{3}{2}x^2+\frac{31}{2}x-\frac{75}{2}$ va

$Y_7(x)=\frac{3}{4}x^2-\frac{37}{4}x+30$;

$$S_{3(i)}(t)=(1-t)Y_i(t)+tY_{i+1}(t),\quad t=(x-x_i)/h;\qquad x\in[x_i,x_{i+1}].$$

Bizga ma'lumki $x_i=0,\quad h=1,\quad x\in[5,6]$. ekanligidan foydalanib

$t = \dfrac{x - x_i}{h} = \dfrac{x - 0}{1} = x$; yani: $t = x$ keltirib chiqaramiz va (9) ga oborib qo‘yamiz. Natijada ushbu ifoda xosil bo‘ladi:

$$S_{3(6)}(x) = (1-x)\cdot Y_6(x) + x\cdot Y_7(x) = (1-x)(-\frac{3}{2}x^2 + \frac{31}{2}x - \frac{75}{2}) + x(\frac{3}{4}x^2 - \frac{37}{4}x + 30) =$$

$$-\frac{3}{2}x^2 + \frac{31}{2}x - \frac{75}{2} + \frac{3}{2}x^3 - \frac{31}{2}x^2 + \frac{75}{2}x + \frac{3}{4}x^3 - \frac{37}{4}x^2 + 30x = S_{3(6)}(x) = \frac{9}{4}x^3 - \frac{105}{4}x^2 + 83x - \frac{75}{2}.$$

$$S_{3(6)}(x) = \frac{9}{4}x^3 - \frac{105}{4}x^2 + 83x - \frac{75}{2}.$$

7. Yuqoridagi kabi qurilgan $Y_7(x)$ va $Y_8(x)$ parabolalarning chiziqli kombinatsiyasi asosida $S_{3(7)}(x)$, $x \in [6,\ 7]$ lokal interpolyatsion kubik splayn funksiya qurildi. Bu erda: $Y_7(x) = \dfrac{3}{4}x^2 - \dfrac{37}{4}x + 30$ va $Y_8(x) = -x^2 + \dfrac{27}{2}x - 43.5$;

$$S_{3(i)}(t) = (1-t)Y_i(t) + tY_{i+1}(t), \quad t = (x - x_i)/h; \quad x \in [x_i, x_{i+1}].$$

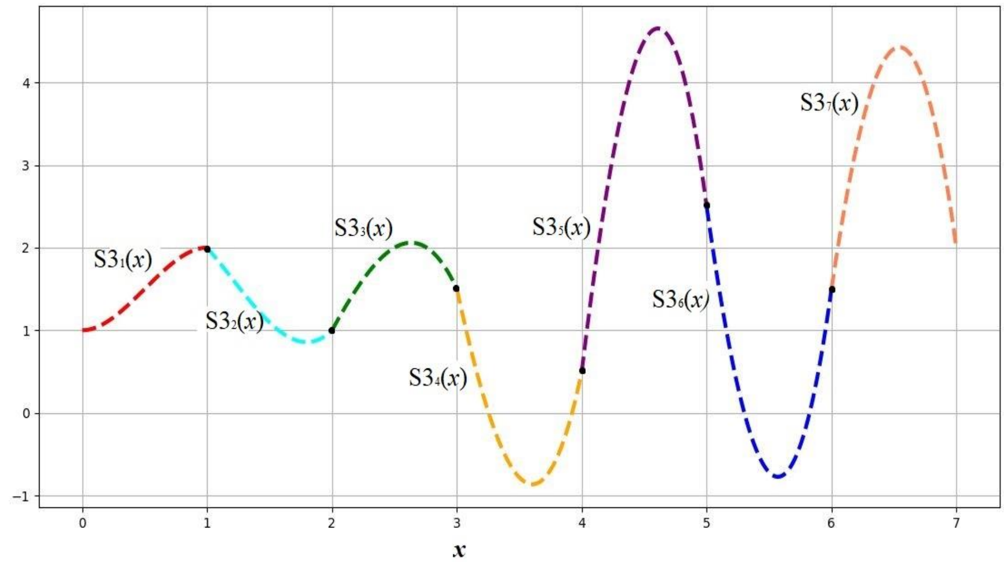
Bizga ma’lumki $x_i = 0$, $h = 1$, $x \in [6, 7]$. ekanligidan foydalanib

$t = \dfrac{x - x_i}{h} = \dfrac{x - 0}{1} = x$; yani: $t = x$ keltirib chiqaramiz va (9) ga oborib qo‘yamiz. Natijada ushbu ifoda xosil bo‘ladi:

$$S_{3(7)}(x) = (1-x)\cdot Y_7(x) + x\cdot Y_8(x) = (1-x)(\frac{3}{4}x^2 - \frac{37}{4}x + 30) + x(-x^2 + \frac{27}{2}x - 43.5) =$$

$$\frac{3}{4}x^2 - \frac{37}{4}x + 30 - \frac{3}{4}x^3 + \frac{37}{4}x^2 - 30x - x^3 + \frac{27}{2}x^2 - 43.5x = -\frac{7}{4}x^3 + \frac{47}{2}x^2 - \frac{331}{4}x + 30.$$

$$S_{3(7)}(x) = -\frac{7}{4}x^3 + \frac{47}{2}x^2 - \frac{331}{4}x + 30.$$



4-rasm.

Yuqorida qurilgan lokal interpolyatsion kubik splayn funksiyalarning grafigi.

Grafikdan ko‘rinib turibdiki splayn funssiyalar quyidagi shartlarni bajaradi:

Ya’ni: ulanish tugun nuqtalarida yonma-yon turgan splaynlarning qiymatlarining tengligini ko‘rish qiyin emas.

$$S_{3(1)}(x_1) = S_{3(2)}(x_1); \qquad S_{3(2)}(x_2) = S_{3(3)}(x_2);$$
$$S_{3(3)}(x_3) = S_{3(4)}(x_3); \qquad S_{3(4)}(x_4) = S_{3(5)}(x_4);$$
$$S_{3(5)}(x_5) = S_{3(6)}(x_5); \qquad S_{3(6)}(x_6) = S_{3(7)}(x_6);$$

Ushbu ishdan olingan natijadan ko‘rinib turibdiki to‘rtta nuqtadan foydalanib qurilgan (ikkita umumiy nuqtaga ega bo‘lgan) parabolalarning chiziqli kombinatsiyalari asosida 7 ta bo‘lakchalarda birlashib [0, 7] oraliqda splayn funksiyani xosil qilinishi tekshirildi hamda oraliqchalarda qurilgan splayn funssiyalarni ulanish tugun nuqtalarida (2 ta tugun nuqta o‘rtasida qurilgan splaynlar) umumiy qiymatga ega ekanligi ham tekshirildi.

Ushbu olingan natijalar o‘quv jarayonida hamda signallarni tiklash va tadbiq qilishda foydalaniladi.

ISSN 3060-4567

FOYDALANILGAN ADABIYOTLAR RO'YXATI:

1. Isroilov M.I. Hisoblash metodlari. 1-qism, Toshkent, O'zbekiston, 2003. 231-330.

2. Алберг Дж., Нильсон Э., Уолш Дж. Теория сплайнов и ее приложения. Москва: Мир, 1972.-316.

3. С.А. Бахрамов, Б.Р.Азимов. Тенг Ораликлар Учун Логранж Ва Локал Нтерполяцион Кубик Сплайн Моделларини Қуриш Ва Сигналларга Тадбиқи. "Ахборот Коммуникация Технологиялари Ва Дастурий Таъминот Яратишда Инновацион Ғоялар" Республика илмий-техник анжуманинг МАЪРУЗАЛАР ТЎПЛАМИ 1- қисм. 55-57 бетлар.ТАТУ, 16-17 апрел 2019 йил.

4. H.N.Zaynidinov, M.A. Kuchkarov, S.A. Baxromov. Geophysical Signals Processing On The Basis Of Bicubic Spline Function. Stemm Abstracts of Uzbek-Israel joint international conference Science – Technology – Education – Mathematics – Medicine. TASHKENT, May 13-15 2019, 167-169 str.

5. H.N.Zaynidinov, B.R. Azimov, S.A. Baxromov. BIOMEDICAL SIGNALS INTERPOLATION CUBIC SPLAIN MODELS. STEMM ABSTRACTS of Uzbek-Israel joint international conference Science – Technology – Education – Mathematics – Medicine. TASHKENT, May 13-15 2019, 169-171 str.