INTERPOLYATSION KUBIK SPLAYNLARNI QURISH

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Annotatsiya: $S(f,x) = S_3(x,f,\Delta_n)$ *funksiya interpolyatsion kubik splayn funksiya deyiladi:*

Har biri $[x_i, x_{i+1}]$ $(i = \overline{0,n})$ *oraliqda* $S(f,x) \in H_3(P)$*;*

$$S(f,x) \in C^2[a,b]$$

To'rning $x_k(k = \overline{0,n})$ *tugunlariga* $S(f,x_k) = f_k$*tenglik o'rinli;*

$S''(f,x)$ *uchun* $S''(f,a) = S''(f,b) = 0$

chegaraviy shartlar bajariladi.

Kalit so'zlar: *splayn funksiyalar, lokal interpolyatsiyalash, local splaynlar, interpolyatsiyon ko'phadlar, klassik interpolyatsiyalash.*

Checking the continuity of spline functions at nodes

Abstract: $S(f,x) = S_3(x,f,\Delta_n)$ The function is called the interpolating cubic spline function:

Each one$[x_i, x_{i+1}]$ $(i = \overline{0,n})$ in between $S(f,x) \in H_3(P)$;

$$S(f,x) \in C^2[a,b]$$

Of the net $x_k(k = \overline{0,n})$ to nodes $S(f,x_k) = f_k$equality holds;

$S''(f,x)$ for $S''(f,a) = S''(f,b) = 0$

Border conditions met.

Keywords: *spline functions, local interpolation, local splines, interpolation polynomials, classic interpolation.*

Interpolyatsion kubik splaynlarni qurish

Jadval ko'rinishida berilgan funksiyalarni splayn funksiyalar bilan yaqinlashtirish. Interpolyatsion kubik splaynlarni qurish.

Ta'rif: Quyidagi to'rt shartni qanoatlantiruvchi ushbu

$S(f,x) = S_3(x,f,\Delta_n)$ funksiya interpolyatsion kubik splayn funksiya deyiladi:

Har biri $[x_i, x_{i+1}]$ $(i = \overline{0,n})$ oraliqda $S(f,x) \in H_3(P)$;

$$S(f,x) \in C^2[a,b]$$

To'rning $x_k(k = \overline{0,n})$ tugunlariga $S(f,x_k) = f_k$tenglik o'rinli;

$S''(f,x)$ uchun $S''(f,a) = S''(f,b) = 0$ (1)

chegaraviy shartlar bajariladi.

Bu to'rt shartni qanoatlantiruvchi yagona $S(f,x)$ splayn mavjudligini ko'rsatamiz. Buning uchun avval quyidagi yordamchi faktlarni keltiramiz.

Endi splaynni qurish bilan shug'illanamiz, $S(f,x)$ ning ikkinchi hosilasi to'rning har biri $[x_{i-1}, x_i]$ oralig'ida uzluksiz bo'lganligi tufayli $x_{i-1} \le x \le x_i$ da ushbu tengsizlikni yoza olamiz:

$$S''(f,x) = M_{i-1}\frac{x_i - x}{h_i} + M_i\frac{x - x_{i-1}}{h_i} \quad (4)$$

Bu yerda $h_i = x_i - x_{i-1}$ va $M_i = S''(f,x_i)$ tenglikning har ikki tomonini integrallab, quyidagiga ega bo'lamiz:

$$S(f,x) = M_{i-1}\frac{(x_i - x)^3}{6h_i} + M_i\frac{(x - x_{i-1})^3}{6h_i} + A_i\frac{x_i - x}{h_i} + B_i\frac{x - x_{i-1}}{h_i} \quad (5)$$

Bunda A_i va $B_i$integrallash doimiylari bo'lib, ular $S(f,x_{i-1}) = f_{i-1}$ va $S(f,x_i) = f_i$ shartlardan aniqlanadi. (5) da $x = x_{i-1},\ x = x_i$ larni o'rniga qo'yib, mos ravishda

$M_{i-1}\frac{h_i^2}{6} + A_i = f_{i-1}$ va $M_i\frac{h_i^2}{6} + B_i = f_i$ larni hosil qilamiz. Bundan A_i va B_i larni topib (4) ga qo'ysak, natijada

$$S(f,x) = M_{i-1}\frac{(x_i - x)^3}{6h_i} + M_i\frac{(x - x_{i-1})^3}{6h_i} +$$

$$+\left(f_{i-1} - \frac{M_{i-1}h_i^2}{6}\right)\frac{x_i - x}{h_i} + \left(f_i - \frac{M_i h_i^2}{6}\right)\frac{x - x_{i-1}}{h_i}, \quad (6)$$

$$S'(f,x) = -M_{i-1}\frac{(x_i - x)^2}{2h_i} + M_i\frac{(x - x_{i-1})^2}{2h_i} + \frac{f_i - f_{i-1}}{h_i} - \frac{M_i - M_{i-1}}{6}h_{i+1} \quad (7)$$

larga ega bo'lamiz.

Oxirgi tenglik $[x_i, x_{i+1}]$ oraliq uchun quyidagi ko'rinishga ega:

$$S'(f,x) = -M_i\frac{(x_{i+1}-x)^2}{2h_{i+1}} + M_{i+1}\frac{(x-x_i)^2}{2h_{i+1}} + \frac{f_{i+1}-f_i}{h_{i+1}} - \frac{M_{i+1}-M_i}{6}h_{i+1} \qquad (8)$$

Endi (7) da x ning x_i ga chapdan va (8) da x ning x_i ga o'ngdan intilgandagi, ya'ni $x_1, x_2, \dots, x_{n-1}$ lar uchun hosilaning bir tomonlama limitlarini hisoblaylik:

$$S'(x_i - 0) = \frac{h_i}{6}M_{i-1} + \frac{h_i}{3}M_i + \frac{f_i - f_{i-1}}{h_i}$$

$$S'(x_i + 0) = -\frac{h_{i+1}}{3}M_i + \frac{h_{i+1}}{6}M_{i+1} + \frac{f_{i+1}-f_i}{h_{i+1}} \qquad (i = \overline{1, n-1})$$

Ta'rifning ikkinchi shartiga ko'ra $S'(f,x)$ va $S''(f,x)$ funksiyalar $[a,b]$ oraliqda uzluksiz.

$S'(f,x)$ ning $x_1, x_2, \dots, x_{n-1}$ nuqtalarda uzluksizligidan foydalansak, quyidagi tenglamaga ega bo'lamiz:

$$\frac{h_i}{6}M_{i-1} + \frac{h_i+h_{i+1}}{3}M_i + \frac{h_{i+1}}{6}M_{i+1} = \frac{f_{i+1}-f_i}{h_{i+1}} - \frac{f_i-f_{i-1}}{h_i} \qquad (9)$$

Bu tenglamaning (1) chegaraviy shartdan kelib chiqadigan

$$M_0 = M_n = 0 \qquad (10)$$

Yuqorida tahlil qilib chiqgan formulalarimizdan foydalanga holda interpolyatsion kubik splayn-funksiya qurishni mo'rib chiqamiz. Bizga quyidagi malum funksiya berilgan bo'lsin va uning tugun nuqtalardagi jadval qiymatlari ham berilsin. Biz interpolyatsiya jarayonining qanchalik yaqinlashishi va qiymatlarning orasidagi farqni solishtirish maqsadida umumiy ko'rinishi mavjud bo'lgan interpolyatsiyalanayotgan funksiyani olamiz.

Biz M larni toppish uchun pragonka usulidan foydalanamiz

$$h_i := xx_i - xx_{i-1}$$

$$a_i := \frac{h_i}{6} \qquad b_i := \frac{h_i + h_{i+1}}{3} \qquad c_i := \frac{h_{i+1}}{6} \qquad d_i := \frac{f_{i+1} - f_i}{h_{i+1}} - \frac{f_i - f_{i-1}}{h_i}$$

$$q_0 \leftarrow 0 \qquad \left| \begin{array}{l} p_i \leftarrow a_i \cdot q_{i-1} + b_i \\ q_i \leftarrow \dfrac{-c_i}{p_i} \\ u_i \leftarrow \dfrac{d_i - a_i \cdot u_{i-1}}{p_i} \end{array} \right.$$
$$u_0 \leftarrow 0$$

$$M_0 := 0 \qquad M_n := 0 \qquad M_{n-1} := u_{n-1}$$

$$k := n - 2..1$$

$$M_k := q_k \cdot M_{k+1} + u_k$$

Misol -1

$$f(x) = x^2$$

i	**0**	**1**	**2**	**3**	**4**
x	**0**	**1**	**2**	**3**	**4**
y	**0**	**1**	**4**	**9**	**16**
	yo	***y1***	***y2***	***y3***	***y4***

Yuqoridagi jadvaldan foydalangan holda (9) tenglamadagi nomalumlarni aniqlab olamiz.

Bunda $\boldsymbol{i} = 1, 2, 3$

$$\frac{h_i}{6} M_{i-1} + \frac{h_i + h_{i+1}}{3} M_i + \frac{h_{i+1}}{6} M_{i+1} = \frac{f_{i+1} - f_i}{h_{i+1}} - \frac{f_i - f_{i-1}}{h_i}$$

$$M_{i-1} + 4 * M_i + M_{i+1} = \frac{6}{h}(y_{i-1} - 2 * y_i + y_{i+1})$$

$\boldsymbol{i = 1}$

ISSN 3060-4567

$$M_0+4*M_1+M_2=6*(y_0-2y_1+y_2)$$
$$4*M_1+M_2=6*(0-2*1+4)$$
$$4*M_1+M_2=12$$

$\boldsymbol{i=2}$

$$M_1+4*M_2+M_3=6*(y_1-2y_2+y_3)$$
$$M_1+4*M_2+M_3=6*(1-2*4+9)$$
$$M_1+4*M_2+M_3=12$$

$\boldsymbol{i=3}$

$$M_2+4*M_3+M_4=6*(y_2-2y_3+y_4)$$
$$M_2+4*M_3=6*(4-2*9+16)$$
$$M_2+4*M_3=12$$

$$\begin{cases} 4*M_1+M_2=12 \\ M_1+4*M_2+M_3=12, \\ M_2+4*M_3=12 \end{cases} \begin{cases} M_1=\dfrac{12-M_2}{4} \\ \Leftarrow \\ M_3=\dfrac{12-M_2}{4} \end{cases},$$

$$\frac{12-M_2}{4}+4*M_2+\frac{12-M_2}{4}=12$$
$$14M_2=48-24,$$
$$M_2=\frac{12}{7},$$
$$M_1=\frac{12-M_2}{4}=\frac{12-\frac{12}{7}}{4}=\frac{84-12}{28}=\frac{18}{7},$$
$$M_3=\frac{12-M_2}{4}=\frac{12-\frac{12}{7}}{4}=\frac{84-12}{28}=\frac{18}{7},$$

Nomalum koeffitsentlarni aniqlab oldik endi kubik splayn-funksiya quramiz

Buning uchun biz:

$$S(f,x)=M_{i-1}\frac{(x_i-x)^3}{6h_i}+M_i\frac{(x-x_{i-1})^3}{6h_i}+$$
$$+\left(f_{i-1}-\frac{M_{i-1}h_i^2}{6}\right)\frac{x_i-x}{h_i}+\left(f_i-\frac{M_ih_i^2}{6}\right)\frac{x-x_{i-1}}{h_i}$$

formulani $\boldsymbol{i=0},1,2,3\ldots$ lar uchun ko’phadlarni yozib chiqamiz.

$\boldsymbol{i=0}$ holat uchun, bizga malumki h=1.

$$S_{3(0)}(x)=M_0\frac{(x_1-x)^3}{6\cdot h_i}+M_1\frac{(x-x_0)^3}{6\cdot h_i}+\frac{x_1-x}{h_i}\left(y_0-\frac{h_i^2}{6}M_0\right)+\frac{x-x_0}{h_i}(y_1-\frac{h_i^2}{6}M_1),$$

$$\Rightarrow\frac{1}{6}\cdot(x-0)^3\cdot\frac{18}{7}+(x-0)(1-\frac{1}{6}\cdot\frac{18}{7})=\frac{3}{7}x^3+\frac{4}{7}x$$

$\boldsymbol{i=1}$

$$S_{3(1)}(x)=M_1\frac{(x_2-x)^3}{6\cdot h_i}+M_2\frac{(x-x_1)^3}{6\cdot h_i}+\frac{x_2-x}{h_i}\left(y_1-\frac{h_i^2}{6}M_1\right)+\frac{x-x_1}{h_i}(y_2-\frac{h_i^2}{6}M_2),$$

$$\Rightarrow\frac{1}{6}\cdot(2-x)^3\cdot\frac{18}{7}+\frac{1}{6}\cdot(x-1)^3\frac{12}{7}+(2-x)\cdot(1-\frac{1}{6}\cdot\frac{18}{7})+(x-1)\cdot(4-\frac{1}{6}\cdot\frac{12}{7})=$$

$$\Rightarrow\frac{3}{7}\cdot(2-x)^3+\frac{2}{7}\cdot(x-1)^3+\frac{4}{7}\cdot(2-x)+\frac{26}{7}\cdot(x-1)$$

$\boldsymbol{i=2}$

$$S_{3(2)}(x)=M_2\frac{(x_3-x)^3}{6\cdot h_i}+M_3\frac{(x-x_2)^3}{6\cdot h_i}+\frac{x_3-x}{h_i}\left(y_2-\frac{h_i^2}{6}M_2\right)+\frac{x-x_2}{h_i}(y_3-\frac{h_i^2}{6}M_3),$$

$$\Rightarrow\frac{1}{6}\cdot(3-x)^3\cdot\frac{12}{7}+\frac{1}{6}\cdot(x-2)^3\cdot\frac{18}{7}+(3-x)\cdot(4-\frac{1}{6}\cdot\frac{12}{7})+(x-2)\cdot(9-\frac{1}{6}\cdot\frac{18}{7})=$$

$$\Rightarrow\frac{2}{7}\cdot(3-x)^3+\frac{3}{7}\cdot(x-2)^3+\frac{26}{7}\cdot(3-x)+\frac{60}{7}\cdot(x-2)$$

$\boldsymbol{i=3}$

$$S_{3(3)}(x)=M_3\frac{(x_4-x)^3}{6\cdot h_i}+M_4\frac{(x-x_3)^3}{6\cdot h_i}+\frac{x_4-x}{h_i}\left(y_3-\frac{h_i^2}{6}M_3\right)+\frac{x-x_3}{h_i}(y_4-\frac{h_i^2}{6}M_4),$$

$$\Rightarrow\frac{1}{6}\cdot(4-x)^3\cdot\frac{18}{7}+\frac{1}{6}\cdot(x-3)^3\cdot 0+(4-x)\cdot(9-\frac{1}{6}\cdot\frac{18}{7})+(x-3)\cdot(16-\frac{1}{6}\cdot 0)=$$

$$\Rightarrow\frac{3}{7}\cdot(4-x)^3+\frac{60}{7}\cdot(4-x)+16\cdot(x-2)$$

$$S_{3i}(x)=\begin{cases}\frac{3}{7}x^3+\frac{4}{7}x,0\le x\le 1\\ \frac{3}{7}\cdot(2-x)^3+\frac{2}{7}\cdot(x-1)^3+\frac{4}{7}\cdot(2-x)+\frac{26}{7}\cdot(x-1),1\le x\le 2\\ \frac{2}{7}\cdot(3-x)^3+\frac{3}{7}\cdot(x-2)^3+\frac{26}{7}\cdot(3-x)+\frac{60}{7}\cdot(x-2),2\le x\le 3\\ \frac{3}{7}\cdot(4-x)^3+\frac{60}{7}\cdot(4-x)+16\cdot(x-2),3\le x\le 4\end{cases}$$

ISSN 3060-4567

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