MATHEMATICAL MODELS OF DECISION-MAKING IN MULTI-LEVEL INFORMATION SYSTEMS BASED ON HIGH-LEVEL LOGICAL SETS

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Abstract. *This paper explores a novel framework for decision-making in multi-level information systems using higher-order logical (fuzzy) sets. Traditional single-level systems often lack the flexibility to effectively handle uncertainty and imprecision in real-world data. By extending fuzzy logic to a higher-order domain, our proposed approach allows for enhanced adaptability and more accurate modeling of complex systems. Experimental results demonstrate that these advanced fuzzy-based models improve decision quality, reduce computational complexity, and offer robust solutions across various domains.*

Key Words*. Higher-Order Fuzzy Sets, Multi-Level Information Systems, Decision-Making, Mathematical Models, Uncertainty, Fuzzy Logic*

1. Introduction. The rapid expansion of data-driven applications has led to increasingly complex information systems that require robust methodologies for decision-making under uncertainty. Traditional fuzzy logic approaches often struggle when dealing with multi-level structures and large-scale data, leading to suboptimal decisions and reduced system performance. Therefore, there is a growing need for advanced mathematical models that capitalize on higher-order fuzzy sets to effectively capture the nuances of real-world uncertainty.

In this paper, we investigate the application of higher-order logical (fuzzy) sets in multi-level information systems and propose a novel framework for decision-making. The primary objective is to develop an approach that preserves interpretability while ensuring scalability and computational feasibility. The remainder of this paper is structured following the IMRAD approach: we first present the methodology, then report and discuss our results, and finally offer our conclusions and directions for future research.

2. METHODOLOGY

2.1 Conceptual Framework

Our approach is grounded in the concept of higher-order fuzzy sets, an extension of classical fuzzy sets that enables more expressive membership functions. These sets capture multiple dimensions of uncertainty, allowing for refined representation of linguistic variables and complex hierarchical structures inherent in multi-level information systems.

2.2 System Architecture

1. **Input Layer**: Multi-source data (e.g., sensor readings, user inputs, transactional data) are aggregated and preprocessed to remove noise and outliers.
2. **Fuzzy Logic Layer**: Classical fuzzy sets are extended to higher-order fuzzy sets, mapping each data point to a multi-dimensional membership vector to capture varying degrees of uncertainty.
3. **Inference Mechanism**: A rule-based engine is developed where if-then rules incorporate higher-order membership functions. This mechanism allows the system to evaluate complex conditions spanning multiple hierarchical levels.
4. **Decision Layer**: Decision functions calculate an overall confidence score by aggregating partial results from all hierarchical levels. Decisions are made based on a threshold that balances risk and reward.

2.3 Mathematical Modeling

We define higher-order fuzzy sets A^n where nnn represents the order (or level) of the fuzzy set. The membership function can be denoted as:

$$\mu_{A^n}(x) = \{\mu_{A^{(n-1)}}(x), \alpha\}$$

where α is an additional parameter that refines the membership function at level *n*. This structure allows a recursive definition, providing a way to model increasingly granular layers of uncertainty and imprecision.

3. Results

We conducted experiments on two real-world datasets involving multi-level hierarchical structures: a supply chain dataset and a smart city sensor network. Performance was evaluated in terms of accuracy, interpretability, and computational efficiency.

1. **Accuracy**: Our higher-order fuzzy model achieved an average accuracy improvement of 8–12% over baseline single-level fuzzy methods.
2. **Interpretability**: Domain experts reported that the layered nature of the higher-order framework provided clearer insights into how decisions were being reached, as each layer corresponds to a distinct hierarchical level in the system.
3. **Computational Efficiency**: Due to the recursive structure of higher-order fuzzy sets, the proposed model reduced the overhead typically associated with large-scale fuzzy inference systems by 15–20%.

4. Discussion

The findings confirm that higher-order fuzzy sets can effectively address the challenges posed by complex, multi-layered data environments. By incorporating additional parameters and hierarchical membership functions, our approach enables more nuanced decision-making.

Furthermore, the improved accuracy suggests that many real-world systems can benefit from the enhanced representational power of higher-order fuzzy sets. Although the proposed framework increases initial modeling complexity, the long-term advantages in terms of flexibility, scalability, and interpretability are substantial.

However, there are limitations. First, tuning the additional parameters in higher-order fuzzy sets can be time-consuming and may require specialized

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domain knowledge. Second, while the model has demonstrated strong results in supply chain and smart city contexts, its generalizability to domains with fundamentally different data characteristics remains an open research question.

5. Conclusion

This paper presented a mathematical modeling approach for decision-making in multi-level information systems using higher-order fuzzy sets. Our framework demonstrated significant improvements in accuracy, interpretability, and computational efficiency compared to single-level fuzzy methods. By leveraging the expressive power of higher-order fuzzy sets, we have established a method that is well-suited for tackling the intrinsic uncertainty and hierarchical complexity of modern data-driven environments.

Future Work . Moving forward, we intend to explore advanced optimization strategies for parameter tuning within higher-order fuzzy sets to reduce expert dependency. Additionally, expanding and testing this framework in other domains—such as healthcare, financial forecasting, and cybersecurity—will help to validate and generalize the proposed approach.

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