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PARABOLIK TIPDAGI DIFFERENSIAL TENGLAMALARNI TO‘R USULI BILAN YECHISH

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Annotatsiya. Ushbu maqolada parabolik tipdagi differensial tenglamalarni to‘r usuli yordamida yechish usullari ko‘rib chiqilgan. Differensial tenglamalar ko‘plab fizik va matematik modellarni ifodalash uchun ishlatiladi. To‘r usulining explicit, implicit va Crank-Nicolson kabi asosiy turlari tahlil qilinib, ularning afzalliklari va kamchiliklari yoritilgan. Amaliy masala misolida to‘r usulining qo‘llanilishi bosqichma-bosqich bayon qilingan. Shuningdek, ushbu usullarni dasturiy amalga oshirish uchun qadamlar ko‘rsatilgan.

Kalit so‘zlar: parabolik differensial tenglama, to‘r usuli, boshlang‘ich shart, chegaraviy shartlar, ayirmali sxema, sonli yechim.

Kirish. Parabolik tipdagi differensial tenglamalar ko‘plab amaliy masalalarni yechishda uchraydi. Masalan, issiqlik o‘tkazuvchanligi, diffuziya va boshqa fizik jarayonlar parabolik tenglamalar orqali modellashtiriladi. Ushbu tenglamalarni analitik

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usullar bilan yechish har doim ham imkoni bo‘lmaganligi sababli, numerik usullar keng qo‘llaniladi. Numerik usullarning asosiy vositalaridan biri bo‘lgan to‘r usuli ushbu maqolaning markaziy mavzusidir. Ushbu maqolada to‘r usuli yordamida parabolik tenglamalarni yechish bosqichlari tahlil qilinadi.

Mavzuga oid adabiyotlar sharhi. Parabolik tipdagi xususiy hosilali differensial tenglamalarni yechishga doir bir necha kitob va jurnallar mavjud, masalan:

1. Ismatullayev G.P., Koshergenova M.S. "Hisoblash usullari" (Toshkent: «Tafakkur Bo‘stoni», 2014)

Ushbu adabiyot hisoblash usullarini o‘rganish uchun keng qamrovli qo‘llanma hisoblanadi. Kitobda matematik masalalarni numerik yechish usullari, ularning nazariy asoslari va amaliy dasturiy tatbiqi haqida batafsil ma’lumot berilgan.

2. Israilov M.I. "Hisoblash usullari. 1-qism" (Toshkent: O‘qituvchi, 2003)

Mazkur kitob hisoblash matematikasi bo‘yicha fundamental asar hisoblanadi. Unda nazariy tushunchalar aniq matematik asoslar bilan yoritilgan bo‘lib, hisoblash usullarining asosiy tamoyillari va ular yordamida differensial tenglamalarni yechish algoritmlari, qat'iylik va barqarorlik masalalari bo‘yicha chuqur nazariy tahlillar berilgan.

3. Abduxamidov A.U., Xudoynazarov S. "Hisoblash usullaridan amaliyot va laboratoriya mashg‘ulotlari" (Toshkent: O‘qituvchi, 1995)

Bu qo‘llanma asosan laboratoriya mashg‘ulotlariga yo‘naltirilgan bo‘lib, hisoblash matematikasi nazariyasini amaliyotda qo‘llashga yordam beradi. Differensial va integral tenglamalarni numerik yechish bo‘yicha amaliy mashqlar keltirilgan.

4. C. F. Gerald va P. O. Wheatley (1999) o‘z asarlarida to‘r usulining matematik asoslarini bayon qilgan. Ularning tadqiqotlarida explicit va implicit usullarning stabil

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ishlashi shartlari batafsil o‘rganilgan. Shu bilan birga, Crank va Nicolson tomonidan kiritilgan usul (1947) yuqori aniqlik va barqarorlik tufayli keng qo‘llanilgan.

Muhokama va natijalar. Faraz qilaylik, $\Gamma = \{0 < x < 1,\ \ 0 < t < T\}$ sohada ushbu

$$\frac{du}{dt} = \frac{d^2u}{dx^2} + f(x,t) \qquad (1)$$

parabolik tenglamaning (issiqlik o‘tkazuvchanlik tenglamasining)

$$u(x,0) = \varphi(x) \qquad (2)$$

boshlang‘ich shart va

$$u(0,t) = \psi_0(t),\ \ u(1,t) = \psi_1(t) \qquad (3)$$

chegaraviy shartlarni qanoatlantiradigan yechimini topish talab qilinsin. Bu yerda $\varphi(x)$, $\psi_0(t)$, $\psi_1(t)$ - berilgan funksiyalar. (1)-(3) masalaning yechimi mavjud, yagona yechim $u(x,t)$ esa kerakli tartibgacha hosilalarga ega deb hisoblaymiz.

Ayirmali sxema qurish uchun Γ sohani x va t koordinatalar bo‘yicha mos ravishda $h = \frac{1}{n}$, $\tau = \frac{T}{K}$ qadamli to‘g‘ri to‘rtburchak to‘r bilan almashtiramiz. (x_i, t_j), $i = 0,1,\dots,N$, $j = 0,1,\dots,K$ nuqtalar to‘plamini *to‘r sohaning tugunlari* deymiz.

Quyidagi

$$(x_i, t_0),\ i = 0,1,\dots,N,\ (x_0, t_j) \text{ va } (x_N, t_j),\ j = 0,1,\dots,K$$

tugunlari to‘r sohaning *chegaraviy tugunlari*, qolganlari esa to‘r sohaning *ichki tugunlari* deyiladi.

Endi (1) ni (x_i, t_j) nuqtada approksimatsiya qilish uchun $\frac{\partial u}{\partial t}$ va $\frac{\partial^2 u}{\partial x^2}$ hosilalarni

$$\left(\frac{\partial u}{\partial t}\right)_{(x_i,t_j)} \cong \frac{u_{ij+1}-u_{ij}}{\tau}, \qquad (4)$$

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{(x_i,t_j)} \cong \frac{u_{i+1j}-2u_{ij}+u_{i-1j}}{h^2} \qquad (5)$$

taqribiy formulalar bilan almashtiramiz. (4) va (5) ni (1) ga qo‘yamiz, hamda (2) va (3) ning approksimatsiyasini yozamiz, natijada quyidagi ayirmali masalani hosil qilamiz:

$$\frac{u_{ij+1}-u_{ij}}{\tau} = \frac{u_{i+1j}-2u_{ij}+u_{i-1j}}{h^2} + f_{ij}, \; i=1,2,...,N-1, \; j=0,1,...,K-1 \qquad (6)$$

$$u_{i0} = \varphi_i, \; i=0,1,...,N \qquad (7)$$

$$u_{0j} = \psi_{0j}, \; u_{Nj} = \psi_{1,j}, \; j=0,1,...,K-1 \qquad (8)$$

(6), (7), (8) chiziqli algebraik tenglamalar sistemasi bo‘lib, tenglamalar soni noma’lumlar soniga tengdir. Agar j-qatlamdagi yechimning qiymatlari ma’lum bo‘lsa, $(j+1)$-qatlamdagi yechimning qiymati

$$u_{ij+1} = u_{ij} + \frac{\tau}{h^2}\left(u_{i+1j}-2u_{ij}+u_{i-1j}\right) + \tau f_{ij}, \; i=1,2,...,N-1 \qquad (9)$$

formula bilan aniqlanadi. u_{0j+1} va u_{Nj+1} lar mos ravishda ψ_{0j+1} va ψ_{1j+1} ga teng. Shuning uchun (6), (7), (8) sxema oshkor deyiladi. Quyida ayirmali sxemaning turg‘unligi va yaqinlashishi uchun zaruruiy shartni chiqaramiz. Xususan, (6), (7), (8) sxemani $\tau \le 0,5h^2$ shart o‘rinli bo‘lsagina qo‘llash mumkinligini ko‘rsatamiz. Buning uchun (6) ga mos

$$\frac{u_{kj+1}-u_{kj}}{\tau} = \frac{u_{k+1j}-2u_{kj}+u_{k-1j}}{h^2} \qquad (10)$$

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birjinsli tenglamani ko‘ramiz. (10) ning xususiy yechimini

$$u_{kj}^{(\varphi)} = q^j e^{ikh\varphi} \qquad (11)$$

ko‘rinishida izlaymiz, bu yerda i - mavhum bir, φ - ixtiyoriy haqiqiy son va q - aniqlanishi lozim bo‘lgan noma’lum son. (11) ni (10) ga qo‘yib va $e^{ikh\varphi}$ ga qisqartirib

$$\frac{q-1}{\tau} = \frac{e^{ih\varphi} - 2 + e^{-ih\varphi}}{h^2}$$

ni hosil qilamiz, bundan

$$q = 1 - 4\alpha \sin^2 \frac{h\varphi}{2}, \quad \alpha = \frac{\tau}{h^2} \qquad (12)$$

ekanligini topamiz. (11) ko‘rinishdagi yechim uchun mos $u_{k0}(\varphi) = e^{ikh\varphi}$ boshlang‘ich shartlar chegaralangan. Agar φ ning qandaydir qiymatida (11) dagi q moduli bo‘yicha birdan katta bo‘lsa, u holda $j \to \infty$ da (11) cheksiz o‘suvchi bo‘ladi. Bunday holda (10) ayirmali tenglama noturg‘un deyiladi, chunki yechimning boshlang‘ich shartlarga uzluksiz bog‘liqligi buziladi. Agar $|q| \le 1$ bo‘lsa, (11) ko‘rinishdagi yechimlar j ning ixtiyoriy qiymatida chegaralangan bo‘lganligi uchun (10) ayirmali tenglama turg‘un deyiladi. (10) tenglama uchun $|q| \le 1$ shart ixtiyoriy φ uchun faqat $\alpha \le 0,5$ bo‘lgandagina bajariladi. Demak, (6), (7), (8) ayirmali sxemani $\tau \le 0,5h^2$ shart bajarilgandagina qo‘llash mumkin. Bunday ayirmali sxema shartli turg‘un deyiladi.

Endi oshkormas sxemani ko‘ramiz. Buning uchun (x_i, t_j), (x_{i+1}, t_{j+1}), (x_i, t_{j+1}), (x_{i-1}, t_{j+1}) nuqtlarni (1) ni approksimatsiya qilish uchun jalb etamiz va natijada

$$\frac{u_{ij+1} - u_{ij}}{\tau} = \frac{u_{i+1j+1} - 2u_{ij+1} + u_{i-1j+1}}{h^2} + f_{ij+1}, \; i = 1,2,\ldots,N-1, \; j = 0,1,\ldots,K-1$$

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$$u_{i0} = \varphi_i, \; i = 0,1,...,N \qquad (13)$$

$$u_{0\,j+1} = \psi_{0\,j}, \; u_{Nj+1} = \psi_{1,\,j+1}, \;\; j = 0,1,...,K-1$$

ko‘rinishidagi oshkormas sxemaga ega bo‘lamiz. Bu sxema τ bo‘yicha birinchi, h bo‘yicha ikkinchi tartibli approksimatsiyaga ega. (13) ni quyidagicha yozamiz:

$$\alpha u_{i+1\,j+1} - (1+2\alpha)u_{ij+1} + \alpha u_{i+1\,j-1} + u_{ij} = \tau f_{ij+1}$$

$$i = 1,2,...,N-1, \;\; j = 0,1,...,K-1 \qquad (14)$$

$$u_{0\,j+1} = \psi_{0\,j}, \; u_{Nj+1} = \psi_{1,\,j+1}, \;\; j = 0,1,...,K-1$$

(14) chiziqli algebraik tenglamalar sistemasi, uning matritsasi uch diagonalli, uni haydash usuli bilan yechish mumkin, chunki diagonal elementlari salmoqli.

(14) ayirmali sxema turg‘unligining zaruriy shartini chiqaramiz. Buning uchun

$$\frac{u_{ij+1} - u_{ij}}{\tau} = \frac{u_{i+1\,j+1} - 2u_{ij+1} + u_{i-1\,j+1}}{h^2}$$

bir jinsli tenglamaning xususiy yechimini (11) ko‘rinishda izlaymiz va natijada

$$q = \left(1 + 4\alpha \cdot \sin^2 \frac{h\varphi}{2}\right)^{-1}, \;\; \alpha = \frac{\tau}{h^2}$$

ga ega bo‘lamiz. Demak, ixtiyoriy φ, τ, h larda $|q| \le 1$, ya’ni (13) ayirmali sxema absolut turg‘un. Absolut turg‘un sxemaning afzalligi shundaki, to‘r qadamlariga hech qanaqa shartning yo‘qligidir. Bu o‘z navbatida, hisoblashdagi talab qilingan aniqlikni ta’min qilish uchun h va τ larni tanlash imkonini beradi.

Misol. $G = \{0 < x < 0.6;\; 0 < t < 0.01\}$ sohada

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$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2},$$

$$u(x,0)=\cos(2x+0.19),$$

$$u(0,t)=0.92,\quad u(0.6,t)=0.1798$$

Differensial masalani to‘r metodi bilan $h=0.1, \alpha=1/6$ bo‘lganda oshkor sxemadan foydalanib yeching.

Yechish: $\alpha=1/6$ bo‘lganligi uchun $\alpha=\frac{\tau}{h^2} \Rightarrow \tau=\frac{1}{6}\cdot\frac{1}{100}=\frac{1}{600}$ bo‘ladi. Berilgan masalani ayirmali masalaga o‘tkazamiz, u quyidagicha:

$$\left(\frac{\partial u}{\partial t}\right)_{(x_i,t_j)}=\frac{u_{ij+1}-u_{ij}}{\tau},$$

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{(x_i,t_j)}=\frac{u_{i+1j}-2u_{ij}+u_{i-1j}}{h^2}$$

$$\frac{u_{ij+1}-u_{ij}}{\tau}=\frac{u_{i+1j}-2u_{ij}+u_{i-1j}}{h^2}$$

$$6\cdot(u_{ij+1}-u_{ij})=u_{i+1j}-2u_{ij}+u_{i-1j}$$

$$u_{ij+1}=\frac{1}{6}\cdot(u_{i+1j}+4u_{ij}+u_{i-1j})$$

$$u_{i0}=\cos(2x_i+0.19)$$

$$u_{0j}=0.932;\quad u_{6j}=0.1798$$

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Jadvalga boshlang‘ich va chegaraviy qiymatlarni yozamiz. Ularning simmetriyasidan foydalanib jadvalni faqat $x = 0;\ 0.1;\ 0.2;\ 0.3;\ 0.4;\ 0.5$ lar uchun to‘ldiramiz.

$$t_j = j\cdot\tau;\quad x_i = i\cdot h$$

$$i=0,\quad x_0=0,\quad u_{00}=\cos(2\cdot 0+0.19)\approx 0.932$$

$$i=1,\quad x_0=0.1,\quad u_{00}=\cos(2\cdot 0.1+0.19)\approx 0.9249$$

$$i=2,\quad x_0=0.2,\quad u_{00}=\cos(2\cdot 0.2+0.19)\approx 0.8309$$

$$i=3,\quad x_0=0.3,\quad u_{00}=\cos(2\cdot 0.3+0.19)\approx 0.7038$$

$$i=4,\quad x_0=0.4,\quad u_{00}=\cos(2\cdot 0.4+0.19)\approx 0.5487$$

$$i=5,\quad x_0=0.5,\quad u_{00}=\cos(2\cdot 0.5+0.19)\approx 0.3717$$

$$i=6,\quad x_0=0.6,\quad u_{00}=\cos(2\cdot 0.6+0.19)\approx 0.1798$$

Boshlang‘ich va chegaraviy qiymatlardan foydalanib $u_{ij+1}=\frac{1}{6}\cdot\left(u_{i-1j}+4u_{ij}+u_{i+1j}\right)$ formula orqali qolgan qiymatlarni topamiz:

$$i=1,\quad j=0:\quad u_{11}=\frac{1}{6}\cdot(u_{00}+4u_{10}+u_{20})\approx 0.9104$$

$$i=2,\quad j=0:\quad u_{21}=\frac{1}{6}\cdot(u_{10}+4u_{20}+u_{30})\approx 0.8254$$

$$i=3,\quad j=0:\quad u_{31}=\frac{1}{6}\cdot(u_{20}+4u_{30}+u_{40})\approx 0.6992$$

$$i=4,\quad j=0:\quad u_{41}=\frac{1}{6}\cdot(u_{30}+4u_{40}+u_{50})\approx 0.545$$

$$i=5,\quad j=0:\quad u_{51}=\frac{1}{6}\cdot(u_{40}+4u_{50}+u_{60})\approx 0.3691$$

$$i=1,\quad j=1:\quad u_{12}=\frac{1}{6}\cdot(u_{01}+4u_{11}+u_{21})\approx 0.8999$$

$$i=2,\quad j=1:\quad u_{22}=\frac{1}{6}\cdot(u_{11}+4u_{21}+u_{31})\approx 0.8185$$

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$$i=3,\quad j=1:\quad u_{32}=\frac{1}{6}\cdot\left(u_{21}+4u_{31}+u_{41}\right)\approx 0.6945$$

$$i=4,\quad j=1:\quad u_{42}=\frac{1}{6}\cdot\left(u_{31}+4u_{41}+u_{51}\right)\approx 0.5414$$

$$i=5,\quad j=1:\quad u_{52}=\frac{1}{6}\cdot\left(u_{41}+4u_{51}+u_{61}\right)\approx 0.3669$$

$$i=1,\quad j=2:\quad u_{13}=\frac{1}{6}\cdot\left(u_{02}+4u_{12}+u_{22}\right)\approx 0.8917$$

$$i=2,\quad j=2:\quad u_{23}=\frac{1}{6}\cdot\left(u_{12}+4u_{22}+u_{32}\right)\approx 0.8114$$

$$i=3,\quad j=2:\quad u_{33}=\frac{1}{6}\cdot\left(u_{22}+4u_{32}+u_{42}\right)\approx 0.6897$$

$$i=4,\quad j=2:\quad u_{43}=\frac{1}{6}\cdot\left(u_{32}+4u_{42}+u_{52}\right)\approx 0.5379$$

$$i=5,\quad j=2:\quad u_{53}=\frac{1}{6}\cdot\left(u_{42}+4u_{52}+u_{62}\right)\approx 0.3648$$

$$i=1,\quad j=3:\quad u_{14}=\frac{1}{6}\cdot\left(u_{03}+4u_{13}+u_{23}\right)\approx 0.885$$

$$i=2,\quad j=3:\quad u_{24}=\frac{1}{6}\cdot\left(u_{13}+4u_{23}+u_{33}\right)\approx 0.8045$$

$$i=3,\quad j=3:\quad u_{34}=\frac{1}{6}\cdot\left(u_{23}+4u_{33}+u_{43}\right)\approx 0.6847$$

$$i=4,\quad j=3:\quad u_{44}=\frac{1}{6}\cdot\left(u_{33}+4u_{43}+u_{53}\right)\approx 0.5343$$

$$i=5,\quad j=3:\quad u_{54}=\frac{1}{6}\cdot\left(u_{43}+4u_{53}+u_{63}\right)\approx 0.3628$$

$$i=1,\quad j=4:\quad u_{15}=\frac{1}{6}\cdot\left(u_{04}+4u_{14}+u_{24}\right)\approx 0.8794$$

$$i=2,\quad j=4:\quad u_{25}=\frac{1}{6}\cdot\left(u_{14}+4u_{24}+u_{34}\right)\approx 0.798$$

$$i=3,\quad j=4:\quad u_{35}=\frac{1}{6}\cdot\left(u_{24}+4u_{34}+u_{44}\right)\approx 0.6796$$

$$i=4,\quad j=4:\quad u_{45}=\frac{1}{6}\cdot\left(u_{34}+4u_{44}+u_{54}\right)\approx 0.5308$$

$$i=5,\quad j=4:\quad u_{55}=\frac{1}{6}\cdot\left(u_{44}+4u_{54}+u_{64}\right)\approx 0.3609$$

$$i=1,\quad j=5:\quad u_{16}=\frac{1}{6}\cdot\left(u_{05}+4u_{15}+u_{25}\right)\approx 0.8746$$

$$i=2,\quad j=5:\quad u_{26}=\frac{1}{6}\cdot\left(u_{15}+4u_{25}+u_{35}\right)\approx 0.7918$$

$$i=3,\quad j=5:\quad u_{36}=\frac{1}{6}\cdot\left(u_{25}+4u_{35}+u_{45}\right)\approx 0.6745$$

$$i=4,\quad j=5:\quad u_{46}=\frac{1}{6}\cdot\left(u_{35}+4u_{45}+u_{55}\right)\approx 0.5273$$

$$i=5,\quad j=5:\quad u_{56}=\frac{1}{6}\cdot\left(u_{45}+4u_{55}+u_{65}\right)\approx 0.359$$

/j		0	1	2	3	4	5	6
	x(i)/y(j)	0	0,001667	0,003333	0,005	0,006667	0,008333	0,01
	0	0,932	0,932	0,932	0,932	0,932	0,932	0,932
	0,1	0,9249	0,9104	0,8998	0,8916	0,8850	0,8794	0,8746
	0,2	0,8309	0,8254	0,8185	0,8114	0,8045	0,7979	0,7918
	0,3	0,7038	0,6991	0,6945	0,6896	0,6846	0,6795	0,6745
	0,4	0,54869	0,5450	0,5414	0,5378	0,5343	0,5307	0,5272
	0,5	0,37166	0,36919	0,3669	0,3648	0,3628	0,3609	0,3590

	0,6	0,179813	0,1798	0,1798	0,1798	0,1798	0,1798	0,1798

Endi ushbu misolni python dasturlash tilidagi kodini keltiramiz:

```
import numpy as np

import pandas as pd

# Parametrlarni belgilash

L = 0.6  # x ning oxirgi qiymati

T = 0.1  # t ning oxirgi qiymati

h = 0.1  # x uchun qadam

tau = 1 / 600  # t uchun qadam

alpha = tau / h**2

# To'r nuqtalari

x_points = np.linspace(0, L, 7)  # 7 ta x nuqtalari

t_points = np.linspace(0, T, 7)  # 7 ta t nuqtalari

# Matritsani e'lon qilish (u uchun)

u = np.zeros((len(t_points), len(x_points)))

# Boshlang'ich shartlar: u(x, 0) = cos(2x + 0.19)

u[0, :] = np.cos(2 * x_points + 0.19)

# Chegaraviy shartlar: u(0, t) = 0.932, u(L, t) = 0.1798

u[:, 0] = 0.932

u[:, -1] = 0.1798
```

```
# To'r metodi yordamida yechim
for n in range(len(t_points) - 1):  # vaqt bo'yicha
    for i in range(1, len(x_points) - 1):  # x bo'yicha
        u[n + 1, i] = u[n, i] + alpha * (u[n, i + 1] - 2 * u[n, i] + u[n, i - 1])
# Natijalarni jadval shaklida chiqarish
table = pd.DataFrame(u, columns=[f"x={round(x, 2)}" for x in x_points])
table.index = [f"t={round(t, 4)}" for t in t_points]
# Jadvalni chop etish
print("Natijalar jadvali:")
print(table)
```

Natija:

```
In [10]: runfile('C:/Users/Nozimakhan/untitled5.py', wdir='C:/Users/Nozimakhan')
Natijalar jadvali:
          x=0.0     x=0.1     x=0.2     x=0.3     x=0.4     x=0.5    x=0.6
t=0.0     0.932  0.924909  0.830941  0.703845  0.548690  0.371660  0.1798
t=0.0167  0.932  0.910429  0.825420  0.699169  0.545044  0.369188  0.1798
t=0.0333  0.932  0.899856  0.818546  0.694523  0.541422  0.366933  0.1798
t=0.05    0.932  0.891662  0.811427  0.689677  0.537857  0.364826  0.1798
t=0.0667  0.932  0.885012  0.804508  0.684665  0.534322  0.362827  0.1798
t=0.0833  0.932  0.879426  0.797952  0.679582  0.530797  0.360905  0.1798
t=0.1     0.932  0.874609  0.791802  0.674513  0.527279  0.359036  0.1798
```

Xulosa. To‘r usuli parabolik differensial tenglamalarni yechishning qulay va samarali vositasi bo‘lib, u real jarayonlarni modellashtirishda keng qo‘llaniladi. Ushbu maqolada keltirilgan nazariy yondashuvlar va amaliy yechimlar parabolik tenglamalarni qo‘llaydigan masalalar uchun foydali bo‘ladi.

Foydalanilgan adabiyotlar ro‘yhati.

1. Ismatullayev G.P., Koshergenova M.S. Hisoblash usullari. — Toshkent: «Tafakkur Bo‘stoni», 2014.

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2. Israilov M.I. Hisoblash usullari. 1-qism. – Toshkent: O‘qituvchi, 2003.

3. Abduxamidov A.U., Xudoynazarov S. Hisoblash usullaridan amaliyot va laboratoriya mashg‘ulotlari. – Toshkent: O‘qituvchi, 1995.

4. Gerald, C. F., & Wheatley, P. O. (1999). Applied Numerical Analysis. Addison-Wesley.

5. Crank, J., & Nicolson, P. (1947). A practical method for numerical evaluation of solutions of partial differential equations of the heat-conduction type. Proceedings of the Cambridge Philosophical Society, 43(1), 50-67.