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GIPERBOLIK TIPDAGI DIFFERENSIAL TENGLAMALARNI TO‘R USULI BILAN YECHISH

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Annotatsiya. Ushbu maqolada giperbolik tipdagi differensial tenglamalarni to‘r usuli yordamida yechish usullari ko‘rib chiqilgan. Giperbolik tipdagi differensial tenglamalar ko‘plab fizik, mexanika va boshqa tabiiy fanlar masalalarini ifodalashda keng qo‘llaniladi.Ushbu maqolada to‘r usulining asosiy tamoyillari, uning afzalliklari va amaliy qo‘llanilishi haqida batafsil ma‘lumot beriladi. To‘r usulining explicit, implicit va Crank-Nicolson kabi asosiy turlari tahlil qilinib, ularning afzalliklari va kamchiliklari yoritilgan. Amaliy masala misolida to‘r usulining qo‘llanilishi bosqichma-bosqich bayon qilingan. Shuningdek, ushbu usullarni dasturiy amalga oshirish uchun qadamlar ko‘rsatilgan.

Kalit so‘zlar: giperbolik differensial tenglama, to‘r usuli, boshlang‘ich shart, chegaraviy shartlar, ayirmali sxema, sonli yechim.

Kirish. Giperbolik tipdagi differensial tenglamalar to‘lqinlar, tebranishlar tarqalishi bilan bog‘liq jarayonlarni matematik modellashtirishda hamda fizika va texnikaning boshqa sohalarida muhim rol o‘ynaydi. Ushbu turdagi muammolar doimiy mexanika, elektrodinamika, akustika, suyuqliklar dinamikasi va termodinamika kabi turli sohalarda paydo bo'ladi. Ushbu tenglamalarni echishning samarali usullaridan biri raqamli usullardan foydalanish bo'lib, ular orasida eng mashhurlaridan biri grid(to‘r) usuli hisoblanadi. Ushbu maqolaning maqsadi giperbolik differensial tenglamalarni grid(to‘r) usuli yordamida yechishning raqamli usullarini o'rganish, shuningdek uni turli sohalarda qo'llash misollarini ko'rib chiqishdir.

Mavzuga oid adabiyotlar sharhi. Giperbolik tipdagi xususiy hosilali differensial tenglamalarni yechishga doir bir necha kitob va jurnallar mavjud, masalan:

1. Ismatullayev G.P., Koshergenova M.S. "Hisoblash usullari" (Toshkent: «Tafakkur Bo‘stoni», 2014)

Ushbu adabiyot hisoblash usullarini o‘rganish uchun keng qamrovli qo‘llanma hisoblanadi. Kitobda matematik masalalarni numerik yechish usullari, ularning nazariy asoslari va amaliy dasturiy tatbiqi haqida batafsil ma’lumot berilgan.

2. Israilov M.I. "Hisoblash usullari. 1-qism" (Toshkent: O‘qituvchi, 2003)

Mazkur kitob hisoblash matematikasi bo‘yicha fundamental asar hisoblanadi. Unda nazariy tushunchalar aniq matematik asoslar bilan yoritilgan bo‘lib, hisoblash usullarining asosiy tamoyillari va ular yordamida differensial tenglamalarni yechish algoritmlari, qat'iylik va barqarorlik masalalari bo‘yicha chuqur nazariy tahlillar berilgan.

3. Abduxamidov A.U., Xudoynazarov S. "Hisoblash usullaridan amaliyot va laboratoriya mashg‘ulotlari" (Toshkent: O‘qituvchi, 1995)

Bu qo‘llanma asosan laboratoriya mashg‘ulotlariga yo‘naltirilgan bo‘lib, hisoblash matematikasi nazariyasini amaliyotda qo‘llashga yordam beradi. Differensial va integral tenglamalarni numerik yechish bo‘yicha amaliy mashqlar keltirilgan.

4. C. F. Gerald va P. O. Wheatley (1999) o‘z asarlarida to‘r usulining matematik asoslarini bayon qilgan. Ularning tadqiqotlarida explicit va implicit usullarning stabil ishlashi shartlari batafsil o‘rganilgan. Shu bilan birga, Crank va Nicolson tomonidan kiritilgan usul (1947) yuqori aniqlik va barqarorlik tufayli keng qo‘llanilgan.

Muhokama va natijalar. Ma'lumki, Koshi masalasi quyidagicha quyiladi: $G=\{t>0,-\infty<x<\infty\}$ sohada ikki marta uzluksiz differensiallanuvchi shunday $u(x,t)$ funksiyani topish kerakki, bu sohada u

$$\frac{d^2u}{dt^2}=\frac{d^2u}{dx^2} \qquad (1)$$

differensial tenglamani qanoatlantirib, t = 0 to‘g‘ri chiziqda

$$u(x,0)=\varphi(x),\ \frac{du}{dt}\Big|_{t=0}=\psi(x) \qquad (2)$$

dastlabki shartlarni qanoatlantirsin, bunda $\varphi(x),\psi(x)$ berilgan funksiyalar.

Differensial tenglamani ayirmali tenglama bilan almashtirish uchun $G_{h\tau}=\omega_h\omega_\tau$ turni kiritamiz, bunda

$$\omega_h=\{x_i=ih,\ i=0,\pm1,\pm2,...,h>0\}$$
$$\omega_\tau=\{t_i=kt,\ k=0,1,2,...,t>0\}$$

keyin 1-chizmadagidek besh nuqtali andazadan foydalanamiz. Bu andaza asosida qurilgan sxema uch qatlamli sxema deyiladi. Bu andazadan quyidagi ayirmali sxema kelib chiqadi:

$$\frac{u_{i,k+1}-2u_{i,k}+u_{i,k-1}}{\tau^2}=\frac{u_{i+1,k}-2u_{i,k}+u_{i-1,k}}{h^2}\ i=1,2,...,\ k=0,1,..., \qquad (3)$$

Chegaraviy shartning ikkinchisini

$$\frac{y_{i,1}-y_{i,0}}{\tau}=\varphi(x_i) \qquad (4)$$

bilan almashtirsak, u holda approksimasiya tartibi $0(\tau)$ bo‘ladi. Ammo chegaraviy shartni ham $0(\tau^2)$ aniqlikda approksimasiya qilish mumkin. Haqiqatan ham,

$$\frac{u(x,\tau)-u(x,0)}{\tau}=\frac{du(x,0)}{dt}+\frac{\tau}{2}\cdot\frac{d^2u(x,0)}{dt^2}+0(\tau^2)$$

yoyilmadan hamda (6. 1) differensial tenglamadan hosil bo‘ladigan

$$\frac{d^2u(x,0)}{dt^2}=\frac{d^2u(x,0)}{dx^2}=\varphi''(x)$$

munosabatdan foydalanib, quyidagiga ega bo'lamiz:

$$\frac{du(x,0)}{dt}=\frac{u(x,\tau)-u(x,0)}{\tau}-\frac{\tau}{2}\varphi''(x)+0(\tau^2)$$

Bundan esa

$$\frac{y_{i,1}-y_{i,0}}{\tau}=\psi(x_i)+\frac{\tau}{2}\varphi''(x_i) \qquad (5)$$

ga ega bo'lamiz. Agar $\varphi(x)$ ning analitik ifodasi berilgan bo'lmasa, u holda $\varphi''(x_i)$ ni $0(h^2)$aniqlikda

$$\Delta_2\varphi_i=\frac{1}{h^2}\left(\varphi(x_{i+1})-2\varphi(x_i)+\varphi(x_{i-1})\right)$$

bilan almashtirish mumkin, natijada

$$\frac{y_{i,1}-y_{i,0}}{\tau}=\psi(x_i)+\frac{\tau}{2}\Delta_2\varphi_i \qquad (6)$$

ga ega bo'lamiz.

Shunday qilib, dastlabki shart, (3) va (6) dan quyidagilarni hosil qilamiz:

$$y_{i,0}=\varphi(x_i),\ y_i'=\varphi(x_i)+\frac{\tau}{2}\Delta_2\varphi_i, \qquad (7)$$

$$y_{i,k+1}=2y_{i,k}+\tau^2\Delta_2 y_{i,k}-y_{i,k-1},\ i=0,\pm1,\pm2,\ldots,\ k=0,1,2,\ldots, \qquad (8)$$

Bundan ko'ramizki, $y_{i,0}$ va y_i' ($i=0,\pm1,\pm2,\ldots,$) qiymatlar (7) dan ma'lum. (8) dan barcha $k=0,1,2,\ldots$ uchun ketma-ket avval $y_{i,2}(0,\pm1,\pm2,\ldots)$, keyin $y_{i,3}(0,\pm1,\pm2,\ldots)$ va h.k. larni topib olinadi.

Endi giperbolik tenglama $\gamma=\frac{\tau}{h}$ uchun qanday shartni bajarish kerakligini tekshiramiz

Faraz qilaylik, ixtiyoriy i va $j\geq 2$ uchun $M(x_i,t_j)$ tugunda $y_{i,j}$ ning qiymatini (8) formula bilan topish kerak bo'lsin. Buning uchun (8) da k = j - 1 deb olib, ko'ramizki, $y_{i,j}$ ning qiymati $y_{i+1,j-1}, y_{i,j-1}, y_{i-1,j-1}$ *va* $y_{i,j-2}$ lar orqali ifodalanadi. Agar $j>3$ bo'lsa, o'z navbatida, $y_{i+1,j-1}, y_{i,j-1}, y_{i-1,j-1}, y_{i,j-2}$ larning qiymatlari past qatlamlardagi $y_{i+2,j-2}, y_{i,j-2}, y_{i-1,j-2}, y_{i-2,j-2}, y_{i+1,j-3}, y_{i,j-3}, y_{i-1,j-3}, y_{i,j-4}$ lar orqali ifodalanadi. Bu jarayonni davom ettirib, oxirgi natijada $y_{i,j}$ ni $y_{m,0}\left(m=i+s, s=0,\pm1,\pm2,...,\pm j-2\right)$ va $y_{m,1}\left(m=i+s, s=0,\pm1,\pm2,...,\pm j-1\right)$ orqali ifodalaymiz. Bu qiymatlarning barchasi teng yonli ΔMCD uchburchak ichida yotadi (2-chizma). Bu uchburchakning uchi $M\left(x_i,t_j\right)$ nuqtada bo'lib, bir tomoni Ox o'qida, qolgan ikki tomoni MS va MD dan iborat. Ular Ox o'qi bilan $\pm arctg\gamma$, $\gamma=\frac{\tau}{h}=const$ burchakni tashkil etadi. MCD uchburchak (8) ayirmali sxemaning aniqlanganlik uchburchagi deyiladi.

Shunday qilib, $y_{i,j}$ ning qiymati M nuqtada (8) tenglama va CD hamda EF kesmalarda yotuvchi $y_{m,0}$ va $y_{m,1}$ dastlabki qiymatlar orqali aniqlanadi. Matematik fizikadan ma'lumki, $u\left(x,t\right)$ yechimning $M\left(x_i,t_j\right)$ nuqtadagi qiymati (1) tenglama hamda $M\left(x_i,t_j\right)$ nuqtadan o'tuvchi

$$t-t_j=x-x_i,\ t-t_j=-x+x_i, \qquad (9)$$

xarakteristikalar t =0 to'g'ri chiziqda ajratadigan kesmadagi shartlar bilan, ya'ni AB kesmadagi boshlang'ich shartlar bilan bir qiymatli ravishda aniqlanadi. (1) tenglamaning (9) xarakteristikalari o'zaro perpendikulyar bo'lib, Ox o'qi bilan $\frac{\pi}{4}$ *va* $\frac{3\pi}{4}$ burchaklarini tashkil etadi; MAB uchburchak (1) differensial tenglamaning aniqlanganlik uchburchagi deyiladi.

Faraz qilaylik, to'rning τ qadami h dan katta bo'lsin (2-chizma). Bu holda $\angle MAB<\angle MCD$ va $tg\left(\angle MCD\right)=\gamma>1$ bo'lib, ayirmali tenglamaning aniqlanganlik uchburchagi differensial tenglamaning aniqlanganlik uchburchagi ichida yotadi.

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Shuning uchun ham CD kesmada beriladigan dastlabki shartlar M nuqtada yechimni aniqlash uchun yetarli emas. Agar AS va DB kesmalarda boshlangich shartlarni o’zgartirilsa, (1), (2) masalaning yechimi butun G sohada, jumladan, M nuqtada o’zgarishi kerak. Ammo $y_{i,j}$ ning to’rdagi qiymati M nuqtada bunday o’zgarishlarga bog’lik, bo’lmasdan, o’zgarmay qoladi. Demak, $\gamma > 1$ bo’lganda (7), (8) ayirmali masalaning yechimi $h \to 0$ da (1), (2) Koshi masalasining yechimiga yaqinlashmaydi; (7), (8) ayirmali masala (1), (2) differensial masalani approksimasiya qilganligi sababli y turg’un bo’la olmaydi, chunki approksimasiya va turg’unlikdan yaqinlashish kelib chiqishi kerak. Bundan shunday xulosaga kelish mumkinki: $\gamma = \frac{\tau}{h} = const$ bo’lganda to’r metodi bilan topilgan taqribiy yechimlar ketma-ketligi $h \to 0$ da yakinlashishi uchun $\gamma \le 1$ shartning bajarilishi zarurdir, ya'ni differensial tenglamaning aniqlanganlik uchburchagi ayirmali tenglamaning aniqlanganlik uchburchagi bilan ustma-ust tushishi yoki uning ichida yotishi kerak. Umumiy holda differensial tenglamaning aniqlanganlik uchburchagi egri chiziqli uchburchak bo’ladi, ammo bu holda ham differensial tenglamaning aniqlanganlik uchburchagi ayirmali sxemaning aniqlanganlik uchburchagi ichida yotishi lozim. Bu shartning bajarilishi uchun to'r qadamlari ma'lum munosabatda olinishi, ya'ni to’rning maxsus tanlanishi talab qilinadi. Differensial tenglamaning koeffisientlaridan va boshlangich shartlaridan ma'lum silliqlik talab qilinganda takribiy yechimlar ketma-ketligining Koshi masalasi yechimiga yaqinlashishi uchun yuqoridagi shart yetarli bo’ladi.

Birinchi chegaraviy masalani yechish. Endi tebranish tenglamasi uchun $G = \{0 < x < 1, 0 < t < T\}$ sohada ushbu birinchi chegaraviy masalani ko’rib chiqamiz. Ya'ni G sohada ikki marta uzluksiz differensiallanuvchi $u(x,t)$ funksiyani topish kerakki, bu sohada u

$$\frac{d^2u}{dt^2} = \frac{d^2u}{dx^2} \qquad (10)$$

tenglamani qanoatlantirib, t= 0 to'g'ri chiziqda

$$u(x,0)=\varphi(x),\ \frac{du}{dt}\Big|_{t=0}=\psi(x) \qquad (11)$$

dastlabki shartlarni va

$$u(0,t)=\mu_1(t),\ u(1,t)=\mu_2(t)\ \ 0\le t\le T \qquad (12)$$

chegaraviy shartlarni qanoatlantirsin. Bu masalani to'r metodi bilan yechish uchun ushbu

$$G_{\tau h}=\left\{x_i=ih, i=\overline{0,M}, hM=1; t_k=k\tau, k=\overline{0,N}, N\tau=T\right\}$$

to'rni kiritamiz va 2-chizmadagidek uch qatlamli andaza bo'yicha (1) differensial tenglamani (3) dagi ayirmali sxema bilan almashtiramiz, bu yerda $i\ va\ k$ quyidagi qiymatlarni qabul qiladi.

$$i=1,2,...,M-1;\ k=1,2,...,N-1$$

Dastlabki shartlar uchun (7) formuladan foydalaniladi. Chegaraviy shartlar quyidagicha yoziladi.

$$y_{0,k+1}=\mu_1(t_{k+1}),\ y_{M,k+1}=\mu_2(t_{k+1})\ \ k=1,2,...,N-1$$

Bularning hammasini birlashtirib, ayirmali sxemaning quyidagi hisoblash algoritmiga ega bo'lamiz:

$$y_{i,0}=\varphi(x_i), y_{i,1}=\varphi(x_i)+\tau\psi(x_i)+\frac{\tau^2}{2}\Delta_2\varphi_i,$$

$$y_{i,k+1}=2y_{i,k}+\tau^2\Delta_2 y_{i,k}-y_{i,k-1},\ i=0,1,2,\ldots,M-1,$$

$$y_{0,k+1}=\mu_1(t_{k+1}),\ y_{M,k+1}=\mu_2(t_{k+1})\ \ k=0,1,2,...,N-1$$

Yuqorida ko'rdikki, bu sxema (1), (3) chegaraviy masalani $0(\tau^2+h^2)$ aniqlikda approksimasiya qiladi. Ko'rsatish mumkinki, agar ixtiyoriy $\varepsilon>0$ uchun $\tau\ va\ h$ qadamlar quyidagi

$$\frac{\tau^2}{h^2}\le\frac{1}{1+\varepsilon}$$

Shartrni qanoatlantirsa, (7),(10),(11) sxema turg'un bo'ladi.

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Misol. $G=\{0\le x\le 1;\ 0\le t\le 0.5\}$ sohada

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2},$$

$$u(x,0)=(1-x^2)\cos(\pi x) \qquad \left.\frac{\partial y}{\partial t}\right|_{t=0}=2x+0.6$$

$$u(0,t)=1+0.4t,\quad u(1,t)=0$$

Differensial masalani to‘r metodi bilan h = l = 0.1 bo‘lganda yeching.

Yechish: Berilgan masalani ayirmali masalaga o‘tkazamiz, u quyidagicha:

$$\left(\frac{\partial^2 u}{\partial t^2}\right)=\frac{u_{ij+1}-2u_{ij}+u_{ij-1}}{\tau^2}$$

$$\left(\frac{\partial^2 u}{\partial x^2}\right)=\frac{u_{i+1j}-2u_{ij}+u_{i-1j}}{h^2}$$

$$\frac{u_{ij+1}-2u_{ij}+u_{ij-1}}{\tau^2}=\frac{u_{i+1j}-2u_{ij}+u_{i-1j}}{h^2}$$

$$u_{ij+1}=u_{i+1j}+u_{i+1j}-u_{ij-1}$$

$$u_{i0}=(1-x_i^2)\cos(\pi x_i)$$

$$u_{0j}=1+0.4t_j;\quad u_{10j}=0$$

$$\frac{u_{i,1}-u_{i,0}}{h}=2x_i+0.6 \qquad u_{i,1}=h\cdot(2x_i+0.6)+u_{i,0}$$

Jadvalga boshlang‘ich va chegaraviy qiymatlami yozamiz. Ulaming simmetriyasidan foydalanib jadvalni faqat x = 0; 0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1; lar uchun to‘ldiramiz.

$$t_j=j\cdot\tau;\quad x_i=i\cdot h$$

Boshlang‘ich va chegaraviy qiymatlarni topamiz:

$$i=0,\quad x_0=0,\quad u_{00}=(1-0^2)\cos(\pi\cdot 0)\approx 1$$

$$i=1,\quad x_1=0.1,\quad u_{10}=(1-0.1^2)\cos(\pi\cdot 0.1)\approx 0.941546$$

$$i=2,\quad x_2=0.2,\quad u_{20}=\left(1-0.2^2\right)\cos\left(\pi\cdot 0.2\right)\approx 0.776656$$

$$i=3,\quad x_3=0.3,\quad u_{30}=\left(1-0.3^2\right)\cos\left(\pi\cdot 0.3\right)\approx 0.534885$$

$$i=4,\quad x_4=0.4,\quad u_{40}=\left(1-0.4^2\right)\cos\left(\pi\cdot 0.4\right)\approx 0.259574$$

$$i=5,\quad x_5=0.5,\quad u_{50}=\left(1-0.5^2\right)\cos\left(\pi\cdot 0.5\right)\approx 0$$

$$i=6,\quad x_6=0.6,\quad u_{60}=\left(1-0.6^2\right)\cos\left(\pi\cdot 0.6\right)\approx -0.19777$$

$$i=7,\quad x_7=0.7,\quad u_{70}=\left(1-0.7^2\right)\cos\left(\pi\cdot 0.7\right)\approx -0.29977$$

$$i=8,\quad x_8=0.8,\quad u_{80}=\left(1-0.8^2\right)\cos\left(\pi\cdot 0.8\right)\approx -0.29125$$

$$i=9,\quad x_9=0.9,\quad u_{90}=\left(1-0.9^2\right)\cos\left(\pi\cdot 0.9\right)\approx -0.1807$$

$$i=10,\quad x_{10}=1,\quad u_{10,0}=\left(1-1^2\right)\cos\left(\pi\cdot 1\right)\approx 0$$

$u_{i,1}=h\cdot\left(2x_i+0.6\right)+u_{i,0}$ ni hisoblaymiz:

$$i=1,\quad x_1=0.1,\quad u_{11}=h\cdot\left(2\cdot x_1+0.6\right)+u_{1,0}\approx 1.021546$$
$$i=2,\quad x_2=0.2,\quad u_{21}=h\cdot\left(2\cdot x_2+0.6\right)+u_{2,0}\approx 0.876656$$
$$i=3,\quad x_3=0.3,\quad u_{31}=h\cdot\left(2\cdot x_3+0.6\right)+u_{3,0}\approx 0.654885$$
$$i=4,\quad x_4=0.4,\quad u_{41}=h\cdot\left(2\cdot x_4+0.6\right)+u_{4,0}\approx 0.399574$$
$$i=5,\quad x_5=0.5,\quad u_{51}=h\cdot\left(2\cdot x_5+0.6\right)+u_{5,0}\approx 0.16$$
$$i=6,\quad x_6=0.6,\quad u_{61}=h\cdot\left(2\cdot x_6+0.6\right)+u_{6,0}\approx -0.01777$$
$$i=7,\quad x_7=0.7,\quad u_{71}=h\cdot\left(2\cdot x_7+0.6\right)+u_{7,0}\approx -0.09977$$
$$i=8,\quad x_8=0.8,\quad u_{81}=h\cdot\left(2\cdot x_8+0.6\right)+u_{8,0}\approx -0.07125$$
$$i=9,\quad x_9=0.9,\quad u_{91}=h\cdot\left(2\cdot x_9+0.6\right)+u_{9,0}\approx 0.059299$$

Endi $u_{0j}=1+0.4t_j;\quad u_{10j}=0$ larni topib olamiz:

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$i=0, j=0, \ u_{0,0}=1+0.4\cdot 0\approx 1$

$i=0, j=1, \ u_{0,1}=1+0.4\cdot 0.1\approx 1.04$

$i=0, j=2, \ u_{0,2}=1+0.4\cdot 0.2\approx 1.08$

$i=0, j=3, \ u_{0,3}=1+0.4\cdot 0.3\approx 1.12$

$i=0, j=4, \ u_{0,4}=1+0.4\cdot 0.4\approx 1.16$

$i=0, j=5, \ u_{0,5}=1+0.4\cdot 0.5\approx 1.2$

Boshlang‘ich va chegaraviy qiymatlardan foydalanib $u_{ij+1}=u_{i+1j}+u_{i+1j}-u_{ij-1}$ formula orqali qolgan qiymatlarni topamiz:

$i=1, \ \ j=1: \ \ u_{12}=\left(u_{21}+u_{01}-u_{10}\right)\approx 0.97511$

$i=2, \ \ j=1: \ \ u_{22}=\left(u_{31}+u_{11}-u_{20}\right)\approx 0.899774$

$i=3, \ \ j=1: \ \ u_{32}=\left(u_{41}+u_{21}-u_{30}\right)\approx 0.741346$

$i=4, \ \ j=1: \ \ u_{42}=\left(u_{51}+u_{31}-u_{40}\right)\approx 0.55531$

$i=5, \ \ j=1: \ \ u_{52}=\left(u_{61}+u_{41}-u_{50}\right)\approx 0.381803$

$i=6, \ \ j=1: \ \ u_{62}=\left(u_{71}+u_{51}-u_{60}\right)\approx 0.2580003$

$i=7, \ \ j=1: \ \ u_{72}=\left(u_{81}+u_{61}-u_{70}\right)\approx 0.210753$

$i=8, \ \ j=1: \ \ u_{82}=\left(u_{91}+u_{71}-u_{80}\right)\approx 0.250775$

$i=9, \ \ j=1: \ \ u_{92}=\left(u_{10,1}+u_{81}-u_{90}\right)\approx 0.109455$

$i=2, \ \ j=2: \ \ u_{23}=\left(u_{32}+u_{12}-u_{21}\right)\approx 0.958228$

$i=3, \ \ j=2: \ \ u_{33}=\left(u_{42}+u_{22}-u_{31}\right)\approx 0.839801$

$i=4, \ \ j=2: \ \ u_{43}=\left(u_{52}+u_{32}-u_{41}\right)\approx 0.8002$

$i=5, \ \ j=2: \ \ u_{53}=\left(u_{62}+u_{42}-u_{51}\right)\approx 0.723575$

$i=6, \ \ j=2: \ \ u_{63}=\left(u_{72}+u_{52}-u_{61}\right)\approx 0.653311$

$i=7, \ \ j=2: \ \ u_{73}=\left(u_{82}+u_{62}-u_{71}\right)\approx 0.610328$

$i=8, \ \ j=2: \ \ u_{83}=\left(u_{92}+u_{72}-u_{81}\right)\approx 0.608546$

$i=9, \ \ j=2: \ \ u_{93}=\left(u_{10,2}+u_{82}-u_{91}\right)\approx 0.391454$

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$$i=1,\quad j=3:\quad u_{14}=\left(u_{23}+u_{03}-u_{12}\right)\approx 0.98469$$
$$i=2,\quad j=3:\quad u_{24}=\left(u_{33}+u_{13}-u_{22}\right)\approx 0.858654$$
$$i=3,\quad j=3:\quad u_{34}=\left(u_{43}+u_{23}-u_{32}\right)\approx 0.822030$$
$$i=4,\quad j=3:\quad u_{44}=\left(u_{53}+u_{33}-u_{42}\right)\approx 0.8982$$
$$i=5,\quad j=3:\quad u_{54}=\left(u_{63}+u_{43}-u_{52}\right)\approx 0.952099$$
$$i=6,\quad j=3:\quad u_{64}=\left(u_{73}+u_{53}-u_{62}\right)\approx 1.003856$$
$$i=7,\quad j=3:\quad u_{74}=\left(u_{83}+u_{63}-u_{72}\right)\approx 0.791028$$
$$i=8,\quad j=3:\quad u_{84}=\left(u_{93}+u_{73}-u_{82}\right)\approx 0.549247$$
$$i=9,\quad j=3:\quad u_{94}=\left(u_{10,3}+u_{83}-u_{92}\right)\approx 0.281999$$

$$i=1,\quad j=4:\quad u_{15}=\left(u_{24}+u_{04}-u_{13}\right)\approx 1.060426$$
$$i=2,\quad j=4:\quad u_{25}=\left(u_{34}+u_{14}-u_{23}\right)\approx 0.966919$$
$$i=3,\quad j=4:\quad u_{35}=\left(u_{44}+u_{24}-u_{33}\right)\approx 0.956654$$
$$i=4,\quad j=4:\quad u_{45}=\left(u_{54}+u_{34}-u_{43}\right)\approx 1.050554$$
$$i=5,\quad j=4:\quad u_{55}=\left(u_{64}+u_{44}-u_{53}\right)\approx 1.248746$$
$$i=6,\quad j=4:\quad u_{65}=\left(u_{74}+u_{54}-u_{63}\right)\approx 1.1328$$
$$i=7,\quad j=4:\quad u_{75}=\left(u_{84}+u_{64}-u_{73}\right)\approx 0.944557$$
$$i=8,\quad j=4:\quad u_{85}=\left(u_{94}+u_{74}-u_{83}\right)\approx 0.681574$$
$$i=9,\quad j=4:\quad u_{95}=\left(u_{10,4}+u_{84}-u_{93}\right)\approx 0.357771$$

Endi ushbu misolni C# dasturlash tilidagi kodini keltiramiz:

```
using System;
using System.Collections.Generic;
using System.Linq;
using System.Text;
using System.Threading.Tasks;
namespace _2_misol
{
    internal class Program
    {
        static void Main(string[] args)
        {
            Console.Write("h qadamni kiriting:");
            double h = double.Parse(Console.ReadLine());
            Console.Write("l qadamni kiriting:");
```

```
double l = double.Parse(Console.ReadLine());
int i, j;
double[,] u = new double[11,6];
double[] xi = new double[11];
double[] ti = new double[6];
for (i = 0; i < 11; i++)
{
   xi[i] = i * h;
   if (i < 6) ti[i] = i * l;
}
for (i = 0; i <= 10; i++)
{
   u[i, 0] = (1+ xi[i]*xi[i])*Math.Cos(Math.PI* xi[i]);
   u[i, 1] = 0.1 * (2*xi[i] + 0.6)+u[i,0];
   if (i <6) { u[0, i] = 1 + 0.4 * ti[i]; }
}
i = 0;
for (i = 0; i < 6; i++)
{
   u[10, i] = 0;
}
for (i = 1; i <= 4; i++)
{//j
   for (j = 1; j <= 9; j++)
   {//i
      u[j, i + 1] = u[j + 1, i] + u[j - 1, i]- u[j, i-1];
   }
}
Console.WriteLine();
Console.WriteLine("--------+-------------+-------------+-------------+-------------+-------------+-------------+-------------
+-------------+-------------+------------+-------------+");
for (i = 0; i < 11; i++)
{
   if (i == 0) Console.Write("ti\\xi \t");
   Console.Write("       "+xi[i] + "   ");
}
Console.WriteLine();
Console.WriteLine("--------+-------------+-------------+-------------+-------------+-------------+-------------+-------------
+-------------+-------------+------------+-------------+");
for (i = 0; i < 6; i++)
{//j
   Console.Write(ti[i] + "\t");
   for (j = 0; j < 11; j++)
```

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```
                    {//i
                        Console.Write("|{0, -13:F7}", u[j, i]);
                    }
                    Console.WriteLine();
                    Console.WriteLine("--------+-------------+-------------+-------------+-------------+-------------+-------------+-------------+-------------+-------------+-------------+-------------+");
                }
            }
        }
    }
```

Natija:

```
qadamni kiriting:0,1
qadamni kiriting:0,1
```

t\xi	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
0	1,0000000	0,9605671	0,8413777	0,6486859	0,3584597	0,0000000	-0,4202631	-0,8758000	-1,3267879	-1,7214123	0,0000000
0,1	1,0400000	1,0405671	0,9413777	0,7686859	0,4984597	0,1600000	-0,2402631	-0,6758000	-1,1067879	-1,4814123	0,0000000
0,2	1,0800000	1,0208106	0,9598753	0,7991515	0,5622262	0,2581966	-0,0955369	-0,4712510	-0,8304244	0,6146244	0,0000000
0,3	1,1200000	0,9993083	0,8785844	0,7614156	0,3588884	0,3066893	0,0272088	-0,2501613	1,2501613	0,6509878	0,0000000
0,4	1,1600000	0,9777738	0,8008485	0,6383213	0,3058787	0,3279005	0,1520649	1,7486211	1,2312510	0,6335369	0,0000000
0,5	1,2000000	0,9615403	0,7375187	0,5453116	0,4073334	0,3512543	2,0493128	1,6334772	1,1339966	0,5802631	0,0000000

Xulosa. To‘r usuli giperbolik differensial tenglamalarni yechishning qulay va samarali vositasi bo‘lib, u real jarayonlarni modellashtirishda keng qo‘llaniladi. Ushbu maqolada keltirilgan nazariy yondashuvlar va amaliy yechimlar giperbolik tenglamalarni qo‘llaydigan masalalar uchun foydali bo‘ladi.

Foydalanilgan adabiyotlar ro‘yhati.

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