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LOG-ALGEBRALARINING BERILISHI

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Annotatsiya: Matematika rivojlanishining zamonaviy bosqichi fan sifatida mukammalikni, hamda zamonaviy olimlarning ilmiy adabiyotlarini jiddiy tahlil etishni talab etadi.

Ushbu muammo bilan F.Sukochev, K. Dykema, D. Zanin, Ayupov Sh.A, Chilin V.I, Abdullayev R.Z, Xudayberganov K.K kabi olimlar shug'ullanmoqda. Xususan, log-algebrasining izometriyasi mavzusi ustida hozirgi kunda bizning mamlakatimizning matematik olimlari hamda ko'pchilik ilmiy tadqiqotchilar shug'ullanmoqda.

Bu mavzuda hali ko'pchilik muammolar o'z yechimini topmagan. Shu sababli, maqola mavzusini log algebrasining berilishi deb nomladik.

Kalit so'zlar: O'lchovli fazo, vector fazo, F-norma, subadditivlik, topologiya, log-algebra.

O'lchovli fazoda (Ω,μ) $L_0 = L_0(\Omega,\mu)$ o'lchovli integrallanuvchi funksiya berilgan bo'lsin. Ω deyarli hamma funksiyada bir xil qiymatda aniqlanadi.

$$L_{\log}(\Omega,\mu) = \{ f \in L_0 \,|\, \int \log(1+|f|)d\mu < +\infty \}.$$

Agar bunda μ cheklangan bo'lsa, $p \in (0,\infty)$ qiymatda $L_p(\Omega,\mu) \subseteq L_{\log}(\Omega,\mu)$ o'rinli bo'ladi. $L_{\log}(\Omega,\mu)$ vektor fazo bo'lsa, uni tengligini tekshirish oson. Har qanday $f \in L_{\log}(\Omega,\mu)$ uchun normasi quyidagiga teng: $\|f\|_{\log} = \int \log(1+|f|)d\mu$

Quyidagi lemma $\|\cdot\|_{\log}$ norma F-norma ekanligini ko'rsatadi.

1-lemma. Barcha $(a)\|f\|_{\log} > 0$ lar uchun $f \neq 0$ qiymatda o'rinli.

$(b)\|\alpha f\|_{\log} \le \|f\|_{\log}$ bo'lsa, barcha f lar va $|\alpha| \le 1$ bo'lganda o'rinli.

$(c)\lim_{\alpha\to 0}\|\alpha f\|_{\log} = 0$ barcha f lar uchun.

$(d)\|f+g\|_{\log} \le \|f\|_{log} + \|g\|_{\log}$.

Isboti. $(a)-(c)$ lar aniq. (d) ni shular orqali isbotlaymiz.

$$\|f+g\|_{log}=\int\log(1+|f+g|)d\mu\le\int\log(1+|f|+|g|+|fg|)d\mu=$$
$$\int(\log(1+|f|)+\log(1+|g|))d\mu=\|f\|_{\log}+\|g\|_{\log}.$$

Bundan ko'rinadiki, $\|f\|_{\log}$ F-norma bo'ladi.

Yuqoridagi lemmadan $d_{\log}(f,g):=\|f-g\|_{\log}$ bo'lishi kelib chiqadi.

$L_{\log}(\Omega,\mu)$ o'zgaruvchi metrik bo'lib, $L_{\log}(\Omega,\mu)$ topologik vector fazo ko'rinishiga keltiriladi.

$L^{\varphi}(\Omega,\mu)$ buni topologiyaga ega deb bilamiz va F-norma quyidagiga teng:

$$\|f\|_{\varphi}:=\inf\{\lambda>0\left\|\frac{|f|}{\lambda}\right\|_{\log}\le\lambda\}$$

2-lemma. $f\in L_{\log}$ bo'lsin.

1) Agar $\|f\|_{\varphi}<1$, bo'lsa, $\|f\|_{\log}\le\|f\|_{\varphi}$ bo'ladi.

2) Agar N butun son bo'lsa, quyidagi o'rinli

$$\|f\|_{\log}\le\frac{1}{N^2}\Rightarrow\|f\|_{\varphi}<\frac{1}{N}.$$

Isboti. Agar $\|f\|_{\log}\le\|f\|_{\varphi}$ bo'lsa, unda har qanday $\lambda<1$ uchun $\left\|\frac{f}{\lambda}\right\|_{\log}\le\lambda$ o'rinli. Demak,

$$\|f\|_{\log}=\int\log(1+|f|)d\mu\le\int\log(1+\frac{|f|}{\lambda})d\mu\le\lambda.$$

λ dan kichikligi birinchini tasdiqlaydi.

Agar N butun son bo'lsa va $\|f\|_{\log}\le\frac{1}{N^2}$ tenglik qo'shish qoidasidan va 3.1.1-lemmadan

$$\int\log(1+N|f|)d\mu=\|Nf\|_{\log}\le N\|f\|_{\log}<\frac{1}{N}$$

kelib chiqib, $\|f\|_{\varphi}<\frac{1}{N}$ tengsizlik o'rinli bo'ladi.

3-lemma. Bizga $f,g,h\in L_{\log}(\Omega,\mu)$ va K haqiqiy musbat son berilgan bo'lsin. U holda $f\cdot g\in L_{\log}(\Omega,\mu)$ bo'lib, quyidagilar o'rinli:

1) $\|Kf\|_{\log}\le\max(K,1)\|f\|_{\log}$,

2) $\|fg\|_{\log}\le\|f\|_{\log}+\|g\|_{\log}$,

3) agar $|g|\le|h|$ bo'lsa, hamma vaqt $\|f\cdot g\|_{\log}\le\|f\cdot h\|_{\log}$ o'rinli.

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Isboti. a va b manfiy emas haqiqiy sonlarni olaylik, u holda biz

$$\log(1+ab) \le \max(\mathrm{b},1)\log(1+\mathrm{a})$$

$$\log(1+ab) \le \max(1+\mathrm{a}) + \log(1+b)$$

larga ega bo'lib, ikkinchini tasdiqlaymiz

$$\|fg\|_{\log} = \int \log(1+|fg|)d\mu \le \int \log((1+|f|+|g|+|fg|)\,d\mu =$$

$$= \int (\log(1+|f|) + \log(1+|g|))d\mu = \|f\|_{\log} + \|g\|_{\log},$$

bundan uchinchi tenglikning to'g'riligi kelib chiqadi.

4-lemma. Takrorlanuvchi $L_{\log}(\Omega,\mu)\times L_{\log}(\Omega,\mu)$ dan $L_{\log}(\Omega,\mu)$ gacha ko'paytma uzluksiz.

Isbot. Biz $\{f_n\}_{n\ge 0}$ va $\{\mathrm{g}_n\}_{n\ge 0}$ ketma-ketliklar $L_{\log}(\Omega,\mu)$ ga yaqinlashuvchi ekanligini ko'rsatishimiz kerak. Mos ravishda $\{f_n g_n\}_{n\ge 0}$ fg ga yaqinlashadi. Qo'shimcha xossadan foydalansak, quyidagi tengsizlik hosil bo'ladi:

$$\|f_n g_n - fg\|_{\log} \le \|(f_n - f)(g_n - g)\|_{\log} + \|f(g_n - g)\|_{\log} + \|(f_n - f)g\|_{\log}.$$

3-lemmadan foydalansak, quyidagi tengsizlikka ega bo'lamiz.

$$\|(f_n - f)(g_n - g)\|_{\log} \le \|f_n - f\|_{\log} + \|(\mathrm{g}_n - g)\|_{\log}$$

va bu yuqori chegarasi nolga teng bo'lib, $n\to\infty$ $K\ge 1$.

$E = \{x\in\Omega \,\|\, f(\mathrm{x})| > K\}$ bo'lsin. Keyin $\|\cdot\|_{\log}$ va 3-lemmaning subadditivligidan foydalangan holda bizda quyidagi tengsizlik hosil bo'ladi.

$$\|f(g_n - g)\|_{\log} \le \|f(g_n - g)\chi E\|_{\log} \le \|K(g_n - g)\|_{\log} + \|f\chi E\|_{\log} + \|g_n - g\|_{\log}$$

$$\le (K+1)\|g_n - g\|_{\log} + \int_{\{|f|\ge K\}} \log(1+|f|)d\mu,$$

Bu yerda, χE E funksiyaning xarakteristik funksiyasi bo'ladi va $\chi E^c = 1-\chi E$. Shunday qilib,

$$\limsup_{n\to\infty}\|f(g_n - g)\|_{\log} \le \int_{\{|f|\ge K\}} \log(1+|f|)d\mu$$

hosil bo'ladi.

Ammo o'ng tomondagi integral 0 dan $K\to\infty$ intiladi, shuning uchun

bizda $\limsup_{n\to\infty}\|(f_n - f)g\|_{\log} = 0$.

Xulosa. $L_{\log}(\Omega,\mu)$ - bu to'liq metrikaga nisbatan topologik algebra, fazo topologiyasi. Shunga mos ravishda bundan keyin $L_{\log}(\Omega,\mu)$ bu topologik algebrani log-algebrasi deymiz.

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